

CS300

Fall 2018, Assignment #4

PROBLEM 3 (2+2+2+2+3P):

- a) Let m denote the *median of 3-median* of N pairwise distinct elements x_1, \dots, x_N from a totally ordered set. Prove that (i) at least $\frac{1}{3}N$ elements are smaller or equal to x_m and (ii) at most $\frac{2}{3}N$ are strictly smaller than x_m . Similarly, prove that the *median of 7-median* m has (iii) at least $\frac{2}{7}N$ elements smaller or equal to x_m and (iv) at most $\frac{5}{7}N$ are strictly smaller.
- b) Consider a variant of the *linear-time median* algorithm, based on the above *median of 3-median*. Set up, justify, and solve an asymptotic recurrence for its running time $T_3(N)$. Repeat for $T_7(N)$. Why do we consider the 5-median instead of 3-median and 7-median?
- c) Draw the Decision Tree for sorting 4 elements a, b, c, d obtained from unrolling BubbleSort according to the lecture.
- d) Draw the Decision Tree for sorting 4 elements a, b, c, d obtained from unrolling MergeSort according to the lecture.
- e) Determine the asymptotic growth of the (solutions of the) following three recurrences:
 - i) $R(n) = 3 \cdot R(0.3 \cdot n) + n$
 - ii) $S(n) = 2 \cdot S(0.3 \cdot n) + S(0.4 \cdot n) + n$
 - iii) $T(n) = 2 \cdot T(0.3 \cdot n) + T(0.5 \cdot n) + n$

Hint: Use an *ansatz* like $c \cdot n \cdot \log n$ or n^c with $c > 0$.

Note that $\varphi(c) := 2 \cdot (0.3)^c + (0.5)^c > 1$ for $c < 1.0984$ and $\varphi(c) < 1$ for $c > 1.0985$.