Martin Ziegler
Donghyun, Jiwon, Sewon, Chansu, Seungwoo, Namjo, Anuar, V.Dung

Issued on Nov.24, 2018
Solutions due: Dec.3

CS300

Fall 2018, Assignment #8

PROBLEM 4 (1+1+1P)**:**
Recall *Knuth-Morris-Pratt* Algorithm for locating matches of pattern $p[0\ldots m-1]$ in $w[0\ldots n-1]$:

$k:=0; j:=0;$ **While** $k<n$ **Do**
  If $w[k]=p[j]$ **Then**
    $k++; j++;$
    If $j=m$ **Then Print** $k-j; j:=T[j];$ **Endif**
  Else $j:=T[j];$ **If** $j<0$ **Then** $k++; j++;$ **Endif**
Endif **Endwhile**

Manually 'preprocess' the following patterns into the auxilary arrays $T[0\ldots m]\in\{-1,0,\ldots m-1\}$:

a) `A B A A B A A A B`

b) `A A A B A A B A B`

c) `A A B A A C A A B A A`

PROBLEM 5 (1+1+1+1P)**:**
Recall that a *flow* in a directed acyclic graph $G=(V,E)$ with weighted adjacency matrix $A\in\mathbb{R}^{V\times V}$ and source/target vertices $s,t\in V$ is a function $f:E\to\mathbb{R}$ satisfying the continuity condition

$$\forall v\in V\setminus\{s,t\}: \quad \sum_{u:(u,v)\in E} f(u,v) \;=\; \sum_{w:(v,w)\in E} f(v,w)$$

The flow is *admissible* if it satisfies $\forall(u,v)\in E:\; f(u,v)\le A_{u,v}$. Let $P=(s=u_0,u_1,\ldots u_{k+1}=t)$ denote a directed path in G, i.e., such that $(u_j,u_{j+1})\in E$ for every $j=0,\ldots k$. Its *unit flow* is $f_P:E\to\mathbb{R}$ with $f_P(u_j,u_{j+1})=1$ for $j=0,\ldots k$; $f_P(v,w)=0$ otherwise.

a) Prove that the set of all flows is a vector space.

b) Prove that the unit flow f_P of a directed path P from s to t is a flow.

d) Prove that the set of all *admissible* flows is convex.

e) If a sequence f_n of admissible flows converges, then $\lim_n f_n$ is again an admissible flow.

PROBLEM 6 (1+1+1P) **:**

a) Let $\Sigma=\{\texttt{0},\texttt{1}\}$ and $V=\{S\}$. Convert the following two rules to Chomsky Normal Form, possibly using additional variables and rules: $S\mapsto\texttt{01}$, $S\mapsto\texttt{0}S\texttt{1}$.
Include some intermediate steps of the conversion process and justify them!

b) Consider $\Sigma=\{\texttt{a},\texttt{b}\}$ and $V=\{S,A,B,T,X\}$ and the following rules:

$$S\mapsto AB,\quad S\mapsto XB,\quad T\mapsto AB,\quad T\mapsto XB,\quad X\mapsto AT,\quad A\mapsto\texttt{a},\quad B\mapsto\texttt{b} \quad (1)$$

Fill a table according to CYK to decide whether the string `aabbb` can be generated by (1).

c) Repeat (b) with the following string: `aaabbb`.