Algorithmic Foundations of Numerics

KAIST CS493 M. Ziegler

Schedule: TuWeTh 12h10—13h, Fr 9h-12h

Place: E3:#3444 **Language:** English **Credit:** 1CP

Instructor: Martin Ziegler, **TA:** 임동현

Attendance: 10 points for missing <3 hours, 9 when missing 3, 8 when missing 4, and so on.

Grading: Homework/Program.assignment 50%, Final exam 40%, Attendance 10%

Homework: Assigned once a week week, to solve over weekend, individual solutions by email

Literature, slides, assignments etc:
`http://kaist.theoryofcomputation.asia/18CS493`

Exam: July 25+26, 12h10~

Background Check

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- Convergent sequence
- Continuous function
- Compact subset
- Metric space
- Logic
- C++
- Unix/Linux
- Halting Problem
- Algorithm design
- and analysis
- Complexity, $\mathcal{NP}$

Theoretical Computer Science *and* Mathematics

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"***Virtues***":

- problem specification
- formal semantics
- algorithm design
- and analysis (correctness, efficiency)
- optimality proof

Reliability in Numerical Software?

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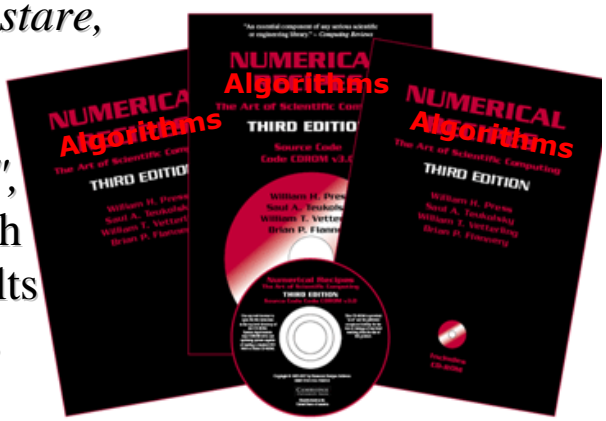
Peter Linz (Courant Institute), p.412, Bull. AMS vol.19:2

«*Over the years, I have sat on many Ph.D. qualifying examinations or dissertation defenses for engineering students whose work involved a significant amount of numerical computing. In one form or another, I invariably ask […]:* "How do you know that your answers are as accurate as you claim?" *[...]*

After an initial blank or hostile stare, I usually get an answer like "I tested the method with some simple examples and it worked", "I repeated the computation with several values of *n* and the results agreed to three decimal places", *or more lamely,* "the answers looked like what I expected".



Specification in Numerics?

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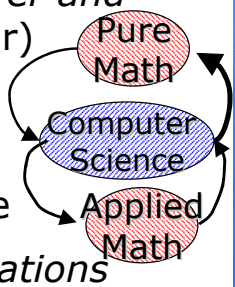
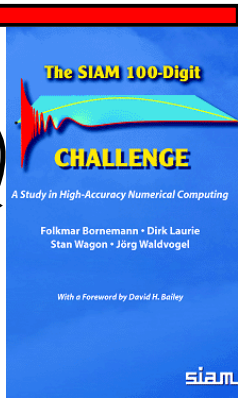
`nag_opt_one_var_deriv (e04bbc)` *normally* computes a sequence of `x` values which *tend in the limit* to a minimum of `F x` subject to the given bounds

- IEEE 754: 5 rounding modes, **NaN**s, *de*normals, **-0** messy to verify [S.Boldo et al.]
- NAG Library, >1700 routines
- MATLAB, • GNU Scientific Library

- Ariane 5 maiden flight
- Sleipner-A oil platform

- W.Tucker (2002): "*Rigorous ODE Solver and Smale's 14th Problem*" (Lorenz Attractor)
- SIAM 100 Digits Challenge (2004)
- T.Hales (Fulkerson Prize 2009): numerical proof of Kepler's conjecture
- D.J. Platt (2013): "*Numerical Computations Concerning the GRH*" and "*Numerical Verification of the Ternary Goldbach Conjecture*" (with H.A.Helfgott).





Debunking Numerical Myths

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Must not test for equality "=="

How about inequality "<" ?

$x=0 \Leftrightarrow \neg(x<0) \wedge \neg(x>0)$

$\rightarrow$ *multivalued* semantics

[Specker'59] There is a computable C^∞ $f:[0;1]\rightarrow[0;1]$ attaining its minimum in no computable point

[Pour-El&Richards'89]
There is a computable initial condition f s.t. solution $u(1)$ is not computable (contains encoding of Halting problem)

$$\Delta u = \ddot{u}$$
$$u(0) = f$$
$$u'(0)=0$$

Weihrauch&Zhong: "*Is Wave Propagation Computable or Can Wave Computers Beat the Turing Machine?*", Proc. London Math. Soc.'02