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CS493

Summer 2018, Assignment #3

PROBLEM 6 (1+1+1+1+1P)**:**
Consider a function $f : [0;1] \to [0;1]$. Recall that a *modulus of continuity* of f is an integer mapping $\mu : \mathbb{N} \to \mathbb{N}$ such that $|x - x'| \leq 2^{-\mu(n)}$ implies $|f(x) - f(x')| \leq 2^{-n}$.

a) Prove that f is Lipschitz-continuous iff it has a modulus of continuity with $\mu(n) \leq n + \mathcal{O}(1)$.

b) Prove that f is Hölder-continuous iff it has a modulus of continuity with $\mu(n) \leq \mathcal{O}(n)$.

c) Plot the function $h : (0;1] \ni t \mapsto 1/\ln(e/t) \in (0;1]$.
Prove that it extends continuously to $h(0) = 0$.

d) Prove that h has an exponential modulus of continuity.

e) Prove that h has no polynomial modulus of continuity.

The *Logistic Map* $f_r : [0;1] \ni x \mapsto r \cdot x \cdot (1-x) \in [0;1]$ is a dynamical system well-known for its chaotic behaviour for many values of the parameter $1 < r < 4$.

PROBLEM 7 (2+1+1+1*+1*+1P)**:**

a) Write, run, and record both output and execution time of a `C++` program printing the

(i) m =30th, (ii) m =40th, (iii) 85th, (iv) 100th, (v) 200th,
(vi) 500th, (vii) 1000th, (viii) 10 000th, (ix) 100 000th, and (x) 500 000th

iterated value $x_{m+1} = f_r(x_m)$ up to six decimal places
for $r := 15/4$ and start value $x_0 := 1/2$ using the data type `float`.

b) Repeat using `double`. Compare.

c) Repeat using `long double` or quadruple precision.

d) Repeat with data type `RATIONAL` after adding these lines to your code:
```
#include "iRRAM.h"
using namespace iRRAM;
```
Also, rename `int main(int argc, char **argv)` to `void compute()`

e) Repeat with data type `REAL`.

f) Explain the above findings!

Feel free to peruse our virtual compute server `irram.theoryofcomputation.asia` accessible via `SSH` and your individual login. A `C++` source file named, say, `logistic.cc` can be compiled, linked, and executed on the shell prompt with the commands `make logistic` and `./logistic`

Alternatively, as root on a linux machine of your choice,
install the library from `http://irram.uni-trier.de`