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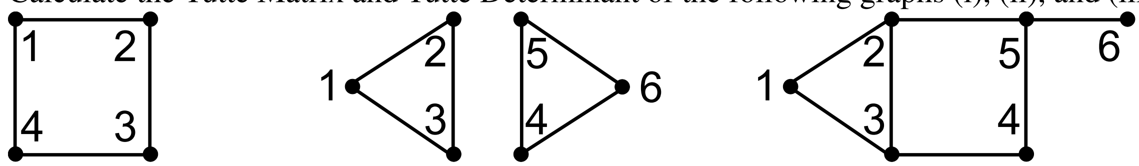
CS500

Spring 2018, Assignment #4

PROBLEM 8 (6+1P) **:**

a) Calculate the Tutte Matrix and Tutte Determinant of the following graphs (i), (ii), and (iii):



b) Estimate the asymptotic number of terms after expanding n-variate polynomial $\prod_{j=1}^{n}(X_j+1)$.

PROBLEM 9 (2+4+2+2P) **:**

a) Implement the randomized (integer) *Polynomial Identity Test* from the lecture in `elice`.

b) Use (a) and Gaussian Elimination, possibly from a library*, to implement in `elice` the randomized algorithm for Perfect Matching in arbitrary graphs with sufficient certainty.

c) Implement in `elice` an algorithm for constructing, given $n, m \in \mathbb{N}$, a graph $G = (V, E)$ with n vertices and an expected number of m undirected edges independently uniformly at random.

d) Implement in `elice` an algorithm creating $N = 50$ random graphs, each with $n = 100$ vertices and $m = 350$ expected edges, and reporting how many of them admit a Perfect Matching; then repeat for $m = 200$.

PROBLEM 10 (1+1+1P) **:**

a) Recall the definition, arithmetic, and properties of quaternions.

b) For 'pure imaginary' quaternions $v = bi + cj + dk$ and $v' = b'i + c'j + d'k$ express the product $v \cdot v'$ in terms of real inner and the cross products.

c) Fix $v = bi + cj + dk$ with $b^2 + c^2 + d^2 = 1$. Establish that the 2D real subspace of the quaternions spanned by 1 and v is closed under multiplication, and is a field isomorphic to the complex numbers.

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*Beware of rounding errors!