Summary

0. Introduction & Motivation

I. Recap of Discrete Computability

II. Computability over the Reals

III. Computability on Topological Spaces

IV. Recap of Discrete Complexity

V. Complexity Theory over the Reals

VI. Complexity on Metric Spaces

VII. Imperative Real Programming

§0 Introduction & Motivation

- "Virtues" of Computer Science <u>and</u> Mathematics
- Data abstraction in Computer Science and Math
- Hardware data types / mathemat. structures
- Programming vs. Software Engineering
- IEEE 754 / Classical Numerics
- Reliability in Numerics, Examples
- Numerical Folklore and Myths

I. Recap of Discrete Computability

- un-/computability, Halting Problem
- Semi-/decidability, Reduction
- Model of computation: WHILE prog.
- SMN propery, Currying
- "Oracle" computation,
  Limit Lemma, Arithmetic Hierarchy

II. Computability over the Reals

a) Computing Real Numbers

- Three equivalent notions,
- counter/examples, oracle-computable reals

b) Computing Real Sequences

- semi-decidability / strong *un*decidability of Equality
- every computable sequence misses a computable Real

c) Computing Real Functions

- closure properties: composition, restriction, sequences
- necessarily continuous
- Computable Weierstrass Theorem
- quantitative continuity

II. Computability over the Reals

d/e) Un/computability with Real Functions

- un/computable Derivative
- un/computable Wave Equation
- un/computable Root Finding

f) Multi-Functions & Enrichment

- generalized restriction, fundamental theorem of algebra
- real computation, fuzzy sign/Heaviside,
- Archimedian property, linear algebra, analytic functions

g) Computing Real Operators

- Encoding continuous functions
- Encoding compact subsets
- Uniform computability
  - Boolean Operations

a) Basic Spaces

- Cantor/Baire Space, Computation
- Complexity, Continuity, Compactness

b) Representations

- Definition, Real Examples revisited
- Realizers, Multi/Functions between Represented Spaces
- (Continuous) Reduction between Representations
- Standard/Admissible representations; *Main Theorem*
- Sequences, Continuous Functions, Compact Subsets

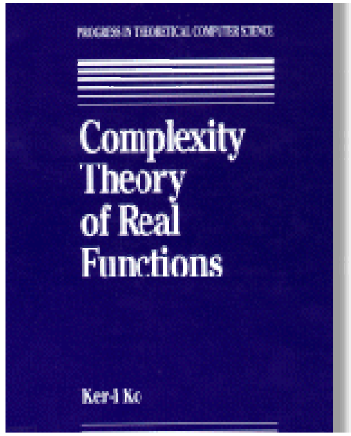
IV. Recap of Discrete Complexity

- Computation with Cost
- Complexity of Arithmetic
- Selected Complexity Classes
- Classifying Example Decision Problems
- Comparing Decision Problems
- Complexity Class Picture

IV. Recap of Advanced Complexity

- Complexity Class Picture
- *Counting* Complexity Class $\#\mathcal{P}$
- *Unary* Complexity Classes
- *Parameterized* Complexity
- *Average*-Case Complexity
- Various Reductions

V. Complexity Theory over the Reals



- *Polytime*-computable real numbers
- *Polytime*-computable real functions
- *Quantitative* computability ⇔ quantitative continuity
- Operations that preserve *polytime* functions
- (Strongly) *polytime*-computable real sequences
- Polytime analytic functions ⇔ polytime Taylor series
- Operations that preserve *polytime* analytic functions

V. Complexity Theory over Reals (2)

- *Parametric* Maximization "*in $\mathcal{NP}$*"
- *Indefinite* Riemann Integration "*in* **#**$\mathcal{P}$"
- *Definite* Riemann Integration "*in* **#**$\mathcal{P}_1$"
- ODESOLVE "*in $\mathcal{PSPACE}$*"
- Parametric Maximization is $\mathcal{NP}$-"*complete*"
- In/definite integration is $\#\mathcal{P}/\#\mathcal{P}_1$-"*complete*"
- Complexity of PDEs: Poisson and Heat Equation
- More numerical characterizations of discrete complexity classes

VI. Complexity on Metric Spaces

- Complexity on (elements of) metric spaces
- Complexity of functions (between metric spaces)
- Complexity and Quantitative Continuity?
- *Polynomial* Admissibility
- Polynomial *Main Theorem*
- Entropy = quantitat. compactness
- *Standard* representation
- 2^{nd}-order Basic Space

qua**l**itative	qua**nt**itative
computability	complexity
topology	metric
(uniformly) continuous	modulus of continuity
compact	entropy
continuous image of compact	*Steinberg's Lemma*
equilogical space	compact *ultrametric*

VII. Imperative Real Programming

- WHILE Programs:
  - over $\mathbb{N}$, over $\mathbb{R}$, over $\mathbb{Z}\,\mathbb{R}\,\mathbb{K}$=Kleene
- *Non*-Extensional Integer Rounding:
  - *naïve* and *fast*
- Exponential Function
- *Tri*section for Simple Unique Root Finding
- Matrix determinant with *multival.* pivoting
- $\mathbb{Z}\mathbb{R}\mathbb{K}$ WHILE Programs: Turing-complete

Computing $f:\subseteq\mathbb{R}\to\mathbb{R}$ means: Convert any $\underline{a}=(a_m)\subseteq\mathbb{Z}$ with $|x-a_m/2^m|\leq 2^{-m}$, to $(b_n)\in\mathbb{Z}$ with $|f(x)-b_n/2^n|\leq 2^{-n}$