"Introduction to Algorithms"

# Syllabus

#### 4. Data

- Hardware/Math Data Types
- Abstract Data Types
- Basic: Boolean, Integer
- Derived: Array, Queue, Stack
- Linked Data Structures
- (Balanced) Search Trees
- AVL Trees

- Specification
- Primitives: semantics and cost
- Design
- Analysis
- (Optimality)

#### Hardware vs. Math. Data Types

- Each country/leader described by "A.I." character parameters.
- Initially Gandhi.aggression := 1
- When country adopts democracy, aggression -= 2.



'Very well, we will mobilize our armies for WAR! You will pay for your foolish pride!' (signed) byte/word/dword ≠ (ℕ) ℤ



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Meta-Def: A structure is a set, together with some

constants,
 functions, and
 relations

Axioms are propositions satisfied by said structure.



Definition: An abstract data type is a "class of objects"together with certain operations.Axioms describe their behaviorregardless of implementation







## **Balanced Binary Trees**





Now insert(15)

and delete(0):

Rebalancing is costly!

Binary **search** tree:

search, insert, delete

in O(h), h = height

## Adelson-Velsky-Landis'62

Binary tree s.t. *left and right subtrees have height difference* <u>*at most 1*</u>!



 $n(h):=\min \ \# \text{nodes of AVLTree of height} \ h \leq O(\log n)$  $\# n(0)+1=F_3, \ \# n(h+1)+1=\# n(h)+1+\# n(h-1)+1 = F_{h+4}$  $\text{Recall } F_h = (\varphi^h - (-1/\varphi)^h)/\sqrt{5} \geq \Omega(1.6^h) \Rightarrow h = O(\log F_h)$ 

## AVL Tree Maintenance

Binary tree s.t. *left and right subtrees have height difference* <u>*at most 1*</u>!

> Store & recursively update balance indicators +, 0, -. After **insert** at a left leaf, propagate up: three cases

 $h \leq O(\log n)$ 

Left Right Case

Left Left Case

Balanced

0(-1)

 $1 \rightarrow 2$ 

search O(log n)
insert O(log n)
delete O(log n)
merge O(n)

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"Introduction to Algorithms"

#### Martin Ziegler

# Recap

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