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CS204

Fall 2017, Assignment #1

Problem 1. *2 + 3 pts*

Let the statement S_n = "Exactly n statements of the set $\{S_1, \ldots, S_{101}\}$ are true".

a) What statements are true/false? Or is there a contradiction?
b) Replace S_n = "At most n statements of the set $\{S_1, \ldots, S_{101}\}$ are true". What happens? Find meaningful results and explain.

Problem 2. *3 + 4 pts*

Recall the definition of the operator **NAND** denoted as $|$. You can see them on Rosen Exercise 1.3. Write corresponding propositions using p, q, r for given truth table

p	0	0	0	0	1	1	1	1
q	0	0	1	1	0	0	1	1
r	0	1	0	1	0	1	0	1
?	1	0	1	1	0	1	0	1

using

a) only binary **AND, OR**, unary **NOT** operators,
b) only binary **NAND** operators.

Use operators as minimal as possible.

Problem 3. *3 + 3 + 2 + 2 + 3 pts*

Recall the Example 16 on the Chapter 1.5. It shows how to make a proposition which is equivalent to $\nexists \lim_{x\to a} f(x)$ where $f : \mathbb{R} \to \mathbb{R}$.

a) Then what is the proposition which corresponds to "For all $a \in \mathbb{R}$, there exists L s.t. $\lim_{x\to a} f(x) = L$"? Find at least 3 equivalent propositions.
b) Real function f is called *continuous* if for every $x \in \mathbb{R}$ and $\epsilon > 0$, there exists $\delta > 0$ such that for every y satisfying $|x - y| < \delta, |f(x) - f(y)| < \epsilon$. Make at least 3 propositions that is equivalent to the definition of *continuous*.
c) Real function f is called *uniformly continuous* if the decision of δ only depends on ϵ, not on x. What changes from b)? Make at least 2 propositions that is equivalent to the definition of *uniforomly continuous*.
d) Suggest a real function that is *continuous* but not *uniformly continuous*.
e) Suggest a real function that is not *continuous* on infinitely many points but the limit exists on every point $x \in \mathbb{R}$.

For a), b), c), don't abuse like just using double negation($\neg\neg$) naively, change order of same quantifiers($\forall x \forall y P \Leftrightarrow \forall y \forall x P$), etc.