Martin Ziegler
Chansu Park, Donghyun Lim, Dongseong Seon

Issued on November 24, 2017
Due: 09:00, December 4, 2017

CS204

Fall 2017, Assignment #8

Problem 1. *5 pts*

Recall the definition of the n-dimensional **hypercube**. It's vertices are represented as $(b_1, \ldots, b_n) \in \{0,1\}^n$. Two points $(b_1, \ldots, b_n), (b'_1, \ldots, b'_n)$ are connected if and only if the only one coordinate of two vertices differs.
Using this representation, prove that every hypercube (whose dimension is 2 or more) has a Hamiltonian Cycle using the induction.

Problem 2. *5 pts*

Suppose that $G = (V, E)$ is a connected simple graph where every vertices in G have even degree. Prove that G can be split into cycles $C_1, \ldots, C_k$ where every two cycle don't share any edge.

Problem 3. *3 pts*

Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$.

Problem 4. *4 + 5 pts*

In graph theory, a **bridge** is an edge of a graph whose deletion increases its number of connected components. Equivalently, an edge is a **bridge** if and only if it is not contained in any cycle.
Also, a set of edges E' is called an **edge cut** of given simple connected graph G if the subgraph $G - E'$ is disconnected. Define $\lambda(G)$ as the minimum number of edges in an edge cut of G.

(a) Suppose that $G = (V, E)$ is a connected simple graph where every vertices in G have even degree. Prove that G contains no bridges.
(b) Suppose that $G = (V, E)$ is a connected simple graph. Prove that if the number of vertices of G is n, then $0 \leq \lambda(G) \leq n - 1$, where the left equality holds if and only if $n = 1$ and the right equality holds if and only if G is a complete graph.

Problem 5. *5 + 2 pts*

There is a sequence of numbers and arithmetic operators which is expressed in a prefix:

$$\times \; / \; + \; 4 \; 5 \; - \; \times \; 3 \; 2 \; 6 \; + \; \times \; 9 \; 0 \; / \; 1 \; - \; 7 \; 8$$

(a) Express it as infix/postfix expression and draw an expression tree.
(b) What is the result of the calculation?

Problem 6. *3 + 4 pts*

Devise a finite automaton for the following languages:

(a) $\{\vec{x} \in \{0,1,2\}^* | \vec{x}$ represents the ternary expansion of an integer divisible by $3\}$
(b) $\{\vec{x} \in \{0,1\}^* | \vec{x}$ represents the binary expansion of an integer divisible by $3\}$

In both item, most significant digit comes first.

Problem 7. *3 + 4 pts*

There is a tree with 26 vertices:

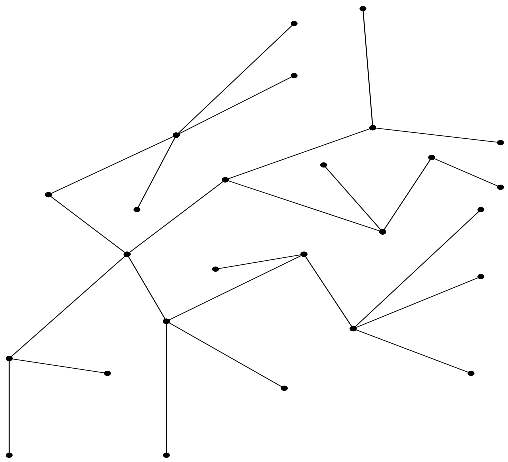

FIGURE 1. An unrooted tree.

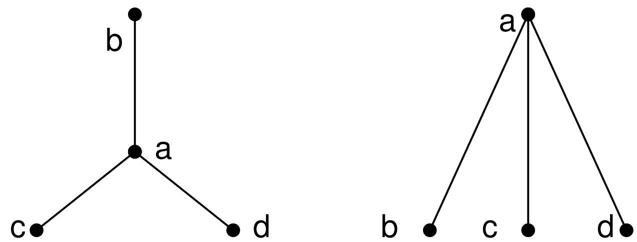

FIGURE 2. An example of unrooted tree (Left) and the rooted tree with level alignment (Right).

(a) Give a root of the tree whose height is minimized. Label the root as a, and then label other vertices from b to z. Then draw your tree where every nodes whose level are the same should be located on the same level. The name of vertices should be increased as the level of each vertices increases.
(b) Perform breadth first search and depth first search to your tree and write the result.

Problem 8. *3 pts*

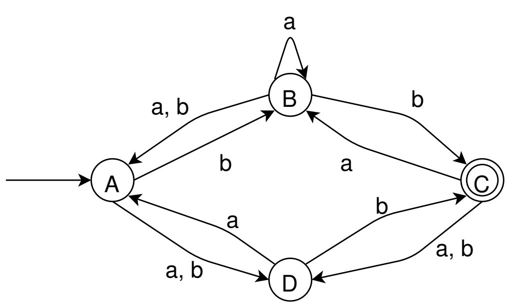

FIGURE 3. A given NFA with $\Sigma = \{a, b\}$.

In a **nondeterministic** finite automaton, the transition function $\delta : S \times \Sigma \to S$ is generalized to a transition "relation" $\Delta \subseteq S \times \Sigma \times S$. When in state $s \in S$ and reading symbol $c \in \Sigma$, the automaton proceeds to "some" new state $t \in S$ satisfying $(s, c, t) \in \Delta$. The automaton **accepts** the word $\bar{w}$ of length $n = |\bar{w}|$ if there exists some sequence $s_0, \ldots, s_n \in S$ of states such that $(s_j, w_{j+1}, s_{j+1}) \in \Delta$ and $s_n \in F$.
Judge wheter or not "bbababba", "abbabab" can be accepted by the given nondeterministic finite automaton.

Problem 9. *4 pts*

Suppose that $L, L' \in \Sigma^*$ are languages which can be accepted by some finite automata A, A', respectively. Prove that both intersection $(A \cap B)$ and union $(A \cup B)$ are regular.