

CS204

Fall 2018, Homework #2

Problem 1.

10 pts

Prove or disprove the following logical equivalences.

- a) $(p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$
- b) $(p \vee r) \wedge (q \vee r) \equiv (p \rightarrow \neg q) \rightarrow r$
- c) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$
- d) $\forall x(P(x) \rightarrow Q(x)) \equiv (\forall xP(x) \rightarrow \forall xQ(x))$
- e) $\neg \exists x \forall y P(x, y) \equiv \forall x \exists y \neg P(x, y)$

Problem 2.

10 pts

Fill the rest of the table to show the truth value of each predicate on each domain \mathbb{N} , \mathbb{Z} , \mathbb{Q} and \mathbb{R} . Explain your answer in each case.

	\mathbb{N}	\mathbb{Z}	\mathbb{Q}	\mathbb{R}
$\forall x(x < 0 \rightarrow x^2 < 0)$				False
$\neg(\forall x \exists y(x + y = 0))$			False	
$\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$				True
$\neg \forall x((x > 0) \rightarrow \exists y(y^2 = x))$	True			

Problem 3.

10 pts

A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded following facts.

- If the butler is telling the truth then so is the cook.
 - The cook and the gardener cannot both be telling the truth.
 - The gardener and the handyman are not both lying.
 - If the handyman is telling the truth then the cook is lying.
- a) Let b , c , g , h be the propositions such that butler, cook, gardener, handyman tells the truth, respectively. Represent the above facts with propositions involving b , c , g , h .
 - b) For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.
 - c) What if the detective knows handyman tells a lie? Now, can the detective determine who are lying and who aren't?