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Total 10 pts

CS204

Fall 2018, Homework #3

Recall the following definitions, where p is an integer and q is a positive integer:

- p is **divisible by** q if p/q is an integer.
- p is **even** if it is divisible by 2. It is **odd** otherwise.
- p is a **prime number** if it is positive and divisible by *exactly* two positive integers.

Problem 1. *4+3+3 pts*

Let n be an integer. Prove the following theorems. (Hint: One method of proof that works is given in brackets after the theorem, but other methods might work.)

a) If n is odd, then $n^2 - 1$ is divisible by 8. (Proof by cases)

b) Let n be a prime number. If k is an integer satisfying $1 < k < n$, then n is *not* divisible by k. (Proof by contraposition)

c) If n is a prime number > 2, then n is odd. (Proof by contradiction)

Problem 2. *2+3+5 pts*

Consider the following expression:

$$(1 \times 2) + (2 \times 5) + (3 \times 8) + (4 \times 11) + \ldots + (n \times (3n - 1))$$

a) Compute the values of the above expression for $n = 1, 2, 3, 4, 5, 6$.

b) Conjecture a formula that is equal to the above expression and is as simple as you can. (Hint: If your answers for part a are correct, the value for the expression at $n = k$ should be divisible by $k + 1$.)

c) Prove your conjecture with mathematical induction.

Problem 3. *5+5 pts*

Goldbach's conjecture states that every even integer ≥ 4 is the sum of **two** prime numbers. For example, $18 = 5 + 13$ is the sum of the two prime numbers 5 and 13. It was proposed by Christian Goldbach in 1742, but so far has not been proven.

In this problem, you will be asked to prove two similar-looking statements.

a) Prove that if Goldbach's conjecture is true, then every integer ≥ 6 is the sum of **three** prime numbers. (Hint: Proof by cases)

b) Prove that every integer ≥ 4 is the sum of **an even number** of prime numbers. Note that this does *not* assume that Goldbach's conjecture is true, so you may not use the result from part a. (Hint: Mathematical induction)