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Total 10 pts

CS204
Fall 2018, Homework #4

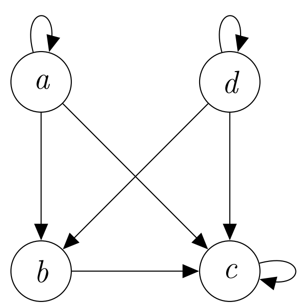
Problem 1. *0.3 × 20 + 2 × 2 pts*

Fill the table with ✓ or ✗. Explain your answers for item b) and d) if your surname starts from A-M. Explain c) and e) otherwise.

a) The binary relation $=$ on $\mathbb{N}$
b) The binary relation $\subseteq$ on $\mathcal{P}(\mathbb{N})$
c) The binary relation $R = \{(r_1, r_2) \in \mathbb{R}^2 \mid |r_1 - r_2| < 0.001\}$ on $\mathbb{R}$
d) The binary relation R on $\{a, b, c, d\}$ represented by the matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

e) The binary relation R on $\{a, b, c, d\}$ represented by the directed graph



	reflexive	symmetric	antisymmetric	transitive
a				
b				
c				
d				
e				

Problem 2. *2 + 6 + 2 pts*

The binary relation R is defined on the set of functions from $\mathbb{Z}^+$ to $\mathbb{Z}^+$. For any functions f, g from $\mathbb{Z}^+$ to $\mathbb{Z}^+$, $(f, g) \in R \Leftrightarrow f = \tilde{\Theta}(g) \Leftrightarrow \exists c\ \{(\lim_{n\to\infty} \frac{f(n)}{g(n)} = c) \wedge (0 < c < \infty)\}$.

a) Prove that for any $f : \mathbb{Z}^+ \to \mathbb{Z}^+$, $f = \tilde{\Theta}(kf)$ where $k \in \mathbb{Z}^+$.
b) Prove that R is an equivalence relation.
c) Prove or disprove that $[n^2 - n + 1]_R = [n^2]_R$.

Problem 3. *4 + 6 pts*

a) Let $A := [0, 1)^2 \subseteq \mathbb{R}^2$. Prove that the poset $(A, \preccurlyeq)$ with the partial order $\preccurlyeq := \{((a, b), (c, d)) \in A \times A \mid a \leq c \wedge b \leq d\}$ has *no* maximal element.

b) Construct a poset that has *infinitely many* maximal elements.