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Total 10 pts

CS204

Fall 2018, Homework #5

Problem 1. *10 pts*

Define the function $f : \mathbb{N} \to \mathbb{N}$ by following.

$$f(n) = \begin{cases} 1, & \text{if } n = 2 \\ 2, & \text{if } n = 1 \\ n, & \text{otherwise} \end{cases}$$

Prove that there is no function $g : \mathbb{N} \to \mathbb{N}$ s.t. $g(g(x)) = f(x)$ for all $x \in \mathbb{N}$.

Problem 2. *10 pts*

Let $f : X \to Y$, $A \subseteq X$ and $B, C \subseteq Y$. For the set B, define preimage as $f^{-1}(B) = \{x | f(x) \in B\}$. Prove the each of followings.

(a) If f is injective, then $f^{-1}(f(A)) = A$
(b) If f is surjective, then $f(f^{-1}(B)) = B$
(c) $f^{-1}(Y \setminus B) = X \setminus f^{-1}(B)$
(d) If f is surjective, then $f^{-1}(B) = f^{-1}(C)$ if and only if $B = C$.

Problem 3. *10 pts*

The Fibonacci sequence F_n is defined as

$$\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_n = F_{n-1} + F_{n-2}, & \text{if } n \geq 2 \end{cases}$$

Prove that

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} \text{ for all } n \geq 1$$