

CS204

Fall 2018, Homework #7

Problem 1.

3 + 3 + 4 pts

How many ways are there, when picking 7 cards (without order) out of a standard French deck (Which has *value* 2-A in 4 *suits*, total 52 cards. *Values* in descending order: A, K, Q, J, 10, 9, ..., 2), of getting (a) four of a kind (*4 cards with the same value*), (b) 10 top (*the highest value among 7 cards is 10*), (c) flush (*5 cards with the same suit*)?

Write your calculation process. Note that it is not needed that given item should be the highest one; for example, even if you have a straight ending at 10, it can be still 10 top.

Problem 2.

3 + 4 + 3 pts

Prove the following identities for all natural numbers $n, r \geq 0$.

$$\text{a) } \sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r} \quad \text{b) } \sum_{k=0}^n k \binom{n}{k} = n2^{n-1} \quad \text{c) } \sum_{k=0}^n \binom{n}{k}^2 = \binom{2n}{n}$$

Problem 3.

3 + 3 + 4 pts

Answer how many ways are there and explain why for each of the items:

- Number of ways to move from $(0, 0)$ to $(4, 4)$ in eight steps, where for each step, from (x, y) you can only go right to $(x + 1, y)$ or go up to $(x, y + 1)$.
- Number of ways to color the sides of a cube with 6 colors so the sides get different colors. (If a coloring can be rotated to another, they are the same.)
- Put 5 indistinguishable objects into 4 indistinguishable boxes