Martin Ziegler, Svetlana Selivanova

Dongseong Seon, Hyunwoo Lee, Ivan Koswara, Seungwoo Schin

Total 10 pts

CS204

Fall 2018, Homework #9

Problem 1. *3+4+3 pts*

a) Monty hall problem is very famous problem of probability theory. Here is the statement of Monty hall problem.
Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?
Of course you want the car instead of goat, so you may want to increase the probability of picking the door where car exists. What is your choice? Answer it using the term of conditional probability and Bayes' theorem.

b) Think about slight different situation. Assume that the host didn't know where is the car. But he pick one door randomly and accidentally get a door with a goat. Does it change your answer of a) or not?

c) Let $F_1, F_2, ..., F_n$ be the mutually exclusive events on sample spaces Ω s.t. $\bigcup_{i=1}^{n} F_i = \Omega$ and $P(F_i) > 0$. For event E with $P(E) > 0$, derive the following.

$$P(F_j|E) = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^{n} P(E|F_i)P(F_i)}$$

Problem 2. *2+3+2+3+3 pts*

Cumulative distribution function of random variable X is defined as

$$F(k) = P(X \leq k)$$

Recall the definition of geometric distribution from the lecture slide, and answer the followings.

a) Let X be a random variable which follows geometric distribution with parameter p. Derive the cumulative distribution function of X.

b) Let $X_1, X_2, \cdots, X_n$ be the independent random variables with geometric distribution with parameter p. Define the random variable Y as $min(X_1, X_2, \cdots, X_n)$. Find the cumulative distribution function of Y.

c) Are the events $X_1 \leq 3$ and $Y \leq 3$ independent?

d) Provide an example of events which are pairwise independent but not mutually independent. (You can use any probability distribution for this item.)

Bonus) I sell the cereal at \$11. Each cereal box contains a special coupon. If you collect every kinds of coupon, you'll get very special gifts. Assume that there are 10 kinds of coupon and you can find each coupon in same probability(i.e. 1/10). Find the expected value of money you spend until you get special gifts.

Problem 3. *2+3+2+3 pts*

Note that $f : \mathbb{N} \to \mathbb{R}$.

a) Prove that $f(n) = e^{-\lambda} \cdot \frac{\lambda^n}{n!}$ for $\lambda \in \mathbb{N}$ is valid probability distribution. This is called Poisson distribution.
b) Prove that mean and variance of Poisson distribution are same.
c) Assume that the number of typo your TA makes in each homework is random variable with mean 5 and variance 1. Let p be the probability that the number of typo in this homework is between 2 and 8. Can p be less than $8/9$?
d) If X is the random variable with mean 0 and variance 1, prove that $P(X > 0) > 0$.