

**CS204**

Fall 2018, Homework #11

**Problem 1.**

3+2+3+2 pts

Prove the followings. Here the definition of tree is connected graph without cycles.

- a) Every tree is a bipartite graph.
- b) Trees with  $n$  vertices have  $(n - 1)$  edges.
- c) Connected graph with  $n$  vertices and  $(n - 1)$  edges is a tree.
- d) Adding an edge to a tree makes it have at least one cycle (i.e. it is not a tree anymore).

**Problem 2.**

2+2+3+3 pts

Recall the binomial tree from the lecture. Similarly, let's define trinomial tree (denoted by  $T_k$ ) recursively. Let  $T_0$  to be a rooted tree with one node (the root), and  $T_k$  to be a rooted tree with three  $T_{k-1}$  combined, by attaching two  $T_{k-1}$ s as a child of the root of one  $T_{k-1}$ .

- a) Draw  $T_2$ .
- b) Give a formula for the height of  $T_k$ . Explain.
- c) Give a formula for the number of nodes of  $T_k$ . Explain.
- d) Give a formula for the degree of the root of  $T_k$ . Explain.