Martin Ziegler *Total 16pts + 5 bonus pts*

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CS204
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Problem 1. *0.5 × 18 pts*

Fill the table with ✓ or ✗ without explanation.

a) The binary relation $=$ on $\mathbb{N}$
b) The binary relation $\subseteq$ on $\mathcal{P}(\mathbb{N})$
c) The binary relation $R = \{(r_1, r_2) \in \mathbb{R}^2 \mid |r_1 - r_2| < 0.001\}$ on $\mathbb{R}$

	reflexive	symmetric	antisymmetric	transitive	total ordering	partial ordering
a						
b						
c						

Problem 2. *2 + 3 pts*

For each conditions, construct a poset $(S, \preccurlyeq)$ which satisfies them.

a) S has *no* minimal element.
b) S has *infinitely many* minimal elements and *no* least element.

For each item, also show why the poset you constructed satisfies the given condition. (You don't have to show it's a poset.)

Problem 3. *2 + 3 + 2 pts*

For any given binary relation $R, S \subseteq A \times A$, define

$$R^n := \begin{cases} \{(a,a)|a \in A\}(= I_A) & n = 0 \\ R & n = 1 \\ R^{n-1} \circ R & n \geq 2 \end{cases}$$

where $R \circ S = \{(a,b) \in A \times A | \exists c \in A : (a,c) \in R \wedge (c,b) \in S\}$.
Also define $R^{-1} := \{(b,a)|(a,b) \in R\}$.
Then define the **reflexive closure of** R as $r(R) := R \cup I_A$, the **symmetric closure of** R as $s(R) := R \cup R^{-1}$, and the **transitive closure of** R as $t(R) := \cup_{n=1}^{\infty} R^n$.
For any given binary relation $R \subseteq A \times A$:

a) Prove that the reflexive closure $r(R)$ is reflexive; and remains symmetric/transitive if R was. Also prove that every reflexive relation containing R also contains $r(R)$ as subset.
b) (Bonus) Prove that the symmetric closure $s(R)$ is symmetric; and remains reflexive if R was. Give a counter example where transitivity is not preserved.
c) (Bonus) Prove that the transitive closure $t(R)$ is transitive; and remains reflexive/symmetric if R was. *Hint: Think about each* $R^n (n \geq 1)$.