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CS204

Spring 2018, Homework #8

Problem 1. *1 + 1 + 1 + 1 pts*

Suppose that we flip 100 coins. Let X_i be a random variable such that it has value 1 if head comes up on i-th coin and 0 otherwise. Compute the following items without explanation (Recall that E denotes expectation and V denotes variance).

a) $E(1 + \sum_{i=1}^{100} 2X_i)$
b) $E((1 + X_1)(1 + X_2))$
c) $V(1 + \sum_{i=1}^{100} 2X_i)$
d) $V((1 + X_1)(1 + X_2))$

Problem 2. *2 + 2 pts*

a) Suppose that there is an experiment with sample space $S = \{a, b, c, d\}$. Assign probabilities and construct three random variables X, Y, Z on this sample space so that they are pairwise independent but not mutually independent. X, Y, Z should have image $\{0, 1\}$.
b) A discrete random variable X with image $\mathbb{N}$ is called **memoryless** if for any $m, n \in \mathbb{N}$, $p(X > m + n | X \geq m) = p(X > n)$ holds. Prove that a random variable X which has geometric distribution with parameter p is memoryless (Here we use slightly different definition from the lecture note. $p(X = k) = (1 - p)^k p$ for $k \in \mathbb{N}$).

Problem 3. *3 + 3 pts*

Suppose that a coin is flipped five times. Let $X(t)$ be the random variable that equals the number of heads that appear when t is the outcome of the coin flips (For example, you can think of $X(t)$ as $X : \{H, T\}^5 \to \{0, 1, 2, 3, 4, 5\}$, $t \in \{H, T\}^5$, and $X(t) = (\textit{number of } H\textit{'s in } t)$. In this case, H denotes head and T denotes tail. For example, $X(H, T, H, H, T) = 3$).

a) Let event $A := \{t \mid X(t) \textit{ is } 1 \textit{ or } 4\}$ and $B := \{t \mid X(t) \geq 3\}$. Are A and B independent? Explain.
b) Let event $C := \{t \mid X(t) \textit{ is even}\}$. Are B and C independent? Explain.