

## CS422

### Spring 2019, Homework #1

#### Problem 1 (6 × 3 points)

For each item below, design a finite automaton<sup>1</sup> that accepts the language described:

- Binary strings that end with 01
- Binary strings with at least 2 digits that begin and end with the same digit
- Binary strings that contain 0011 somewhere in it
- Binary strings, where if we interpret them as a binary number<sup>2</sup>, is divisible by 2
- Binary strings, where if we interpret them as a binary number, is divisible by 3
- The set  $\{0^m 1^n \mid m, n \in \mathbb{N}\}$

#### Problem 2 (2 + 4 + 1 + 5 + 5 points)

For a fixed integer  $n$ , consider the following model of computation:

- The input is a sequence of  $n$  distinct integers  $(a_1, a_2, \dots, a_n)$ .
- A program is a sequence of commands. A command is one of the following:
  - "If  $a_i > a_j$ , swap  $a_i$  with  $a_j$ , otherwise do nothing", for some  $1 \leq i, j \leq n$
  - "Output the value of  $a_i$ ", for some  $1 \leq i \leq n$
- A program is executed on an input by running each command in sequence, from the first to the last command. The output of the execution is the sequence of values printed by the output commands.

As an example, let  $n = 2$ . The following is a valid program:

- If  $a_1 > a_2$ , swap  $a_1$  with  $a_2$ , otherwise do nothing.
- Output the value of  $a_1$ .
- Output the value of  $a_2$ .

Solve the following items:

- Execute the above example program for input  $(2, 4)$  and for input  $(4, 2)$ .
- Let  $n = 4$ . Write a program (in this model of computation) that prints the input but sorted in increasing order.
- Same as item b, but your program should have at most 9 commands.
- Prove that, for any  $n$ , there exists a program<sup>3</sup> that prints the maximum of the input.
- Prove that, for any  $n$ , there exists a program that prints the input but sorted in increasing order.

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<sup>1</sup> If you are familiar with the various automata, here we only consider deterministic finite automata; your automata cannot be nondeterministic. If you are not familiar with them, feel free to ignore this note and just follow the examples in the lecture.

<sup>2</sup> Strings may contain leading zeroes and may be empty. For example, the empty string, the string 0, and the string 00 all evaluate to the number 0, and the string 010 evaluates to the number 2.

<sup>3</sup> The program will necessarily be different for different  $n$ .