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CS422

Spring 2019, Homework #5

Problem 14 (5×10 points)

Define the following decision problems. Red text indicates an example.

SET-COVER: Let F be a family of subsets of $\{1, 2, \dots, n\}$. For example, when $n = 4$, F might be $\{\{1,2\}, \{1,3\}, \{2,3,4\}\}$. Given F, n, and a natural number k, is it possible to choose at most k sets from F such that their union is $\{1, 2, \dots, n\}$? For the above example, the answer is yes when $k = 2$ (pick $\{1,2\}$ and $\{2,3,4\}$), but no when $k = 1$.

HITTING-SET: Let F be a family of subsets of $\{1, 2, \dots, n\}$. Given F, n, and a natural number k, is it possible to choose at most k elements of $\{1, 2, \dots, n\}$ such that every set in F has at least one of the chosen elements? For the above example, the answer is yes when $k = 2$ (pick 1 and 2), but no when $k = 1$.

DOMINATING-SET: Given a graph G and a natural number k, is it possible to choose at most k vertices of G such that every vertex not chosen is adjacent to at least one of the chosen vertices? For the 3×3 rook graph defined in Homework 4, Problem 10, the answer is yes when $k = 3$ (pick vertices along the first row), but no when $k = 2$.

VC (vertex cover) was defined in the lecture on graph problems. **TAUTOLOGY** and **EQUIV** (equivalence) were defined in the lecture on Boolean formulas. **SUDOKU** was defined in Homework 4, Problem 12.

Prove each statement below by constructing an explicit polynomial-time reduction (without going through another decision problem). Remember to prove that your construction satisfies the properties of a polynomial-time reduction. Some hints for the recommended construction are given, but you may construct any reduction (without necessarily using the hint) as long as it's correct. If you are unable to come up with some reduction, just write down ideas you have, what you think the reduction will do.

a. TAUTOLOGY $\leq_p$ EQUIV and EQUIV $\leq_p$ TAUTOLOGY (**Hint:** Remember Homework 2.)
b. VC $\leq_p$ SET-COVER (**Hint:** How is SET-COVER similar to VC?)
c. SET-COVER $\leq_p$ HITTING-SET and HITTING-SET $\leq_p$ SET-COVER (**Hint:** Draw a table. The rows are $1, 2, \dots, n$, and the columns are the sets in F.)
d. SUDOKU $\leq_p$ HITTING-SET (**Hint:** Each possible cell-content combination is an element.)
e. (Bonus) SET-COVER $\leq_p$ DOMINATING-SET (**Hint:** Make a bipartite graph. Each vertex on the right side has $k + 1$ identical copies.)

Problem 15 (10 points)

Write a nondeterministic program (using the "guess" instruction) that runs in polynomial time and decides DOMINATING-SET, thus showing this is in NP. You may use pseudocode; the important part is that you understand the "guess" instruction. Additionally, prove that your program works.