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CS422

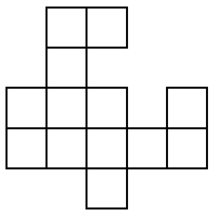
Spring 2019, Homework #6

Problem 16 (6 + 6 + 6 points)

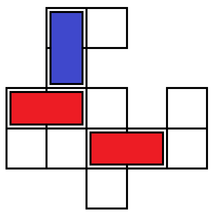
Recall that Cook-Levin theorem states that SAT is **NP**-complete. In this problem, you may use this theorem without proof.

a. Prove that if L is **NP**-complete, then L^c is **coNP**-complete, and vice versa.
b. Recall the definition of TAUTOLOGY in the Boolean formulas lecture. Prove that TAUTOLOGY is **coNP**-complete.
c. Prove that if there exists a problem that is both **NP**-complete and **coNP**-complete, then **NP** = **coNP**, and vice versa.

Problem 17 (4 + 4 + 6 + 6 points)

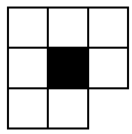


A polyomino is a set of unit squares attached edge to edge. For example, the picture to the right is a polyomino made of 13 unit squares. As another example, the pieces in the game Tetris are polyominoes.



The game Domineering is played on a polyomino. The two players are named Horizontal and Vertical; turns alternate between them. On a player's turn, he places a 1x2 domino so that it is fully inside the polyomino and doesn't cover any existing domino. The difference is the orientation: Horizontal must only play dominoes oriented horizontally while Vertical must only play dominoes oriented vertically. The game continues until the player about to move is unable to place any domino. In that case, that player loses.

The picture to the right is an example of a game in progress. Horizontal was the starting player. It's now Vertical's turn, and there is only one move available (far right column). Horizontal will then take his only move too (left part), and then Vertical cannot play and so Vertical loses.



a. Prove that any game of Domineering must always end.
b. Consider the polyomino to the right (it is made of 7 squares, with a hole in the middle), and suppose Horizontal starts. Determine which player wins assuming both player play optimally, and prove your claim.

Let DOMINEERING-H be the following decision problem: given a polyomino P and the first player, and that Horizontal and Vertical play Domineering on P, does Horizontal win assuming both players play optimally? Let DOMINEERING-V be defined similarly but asking whether Vertical wins instead.

c. Prove that DOMINEERING-H $\leq_p$ DOMINEERING-V and vice versa.
d. Prove that DOMINEERING-H is in **PSPACE**.

Problem 18 (0 points)

What is the most memorable thing you learned from this course?