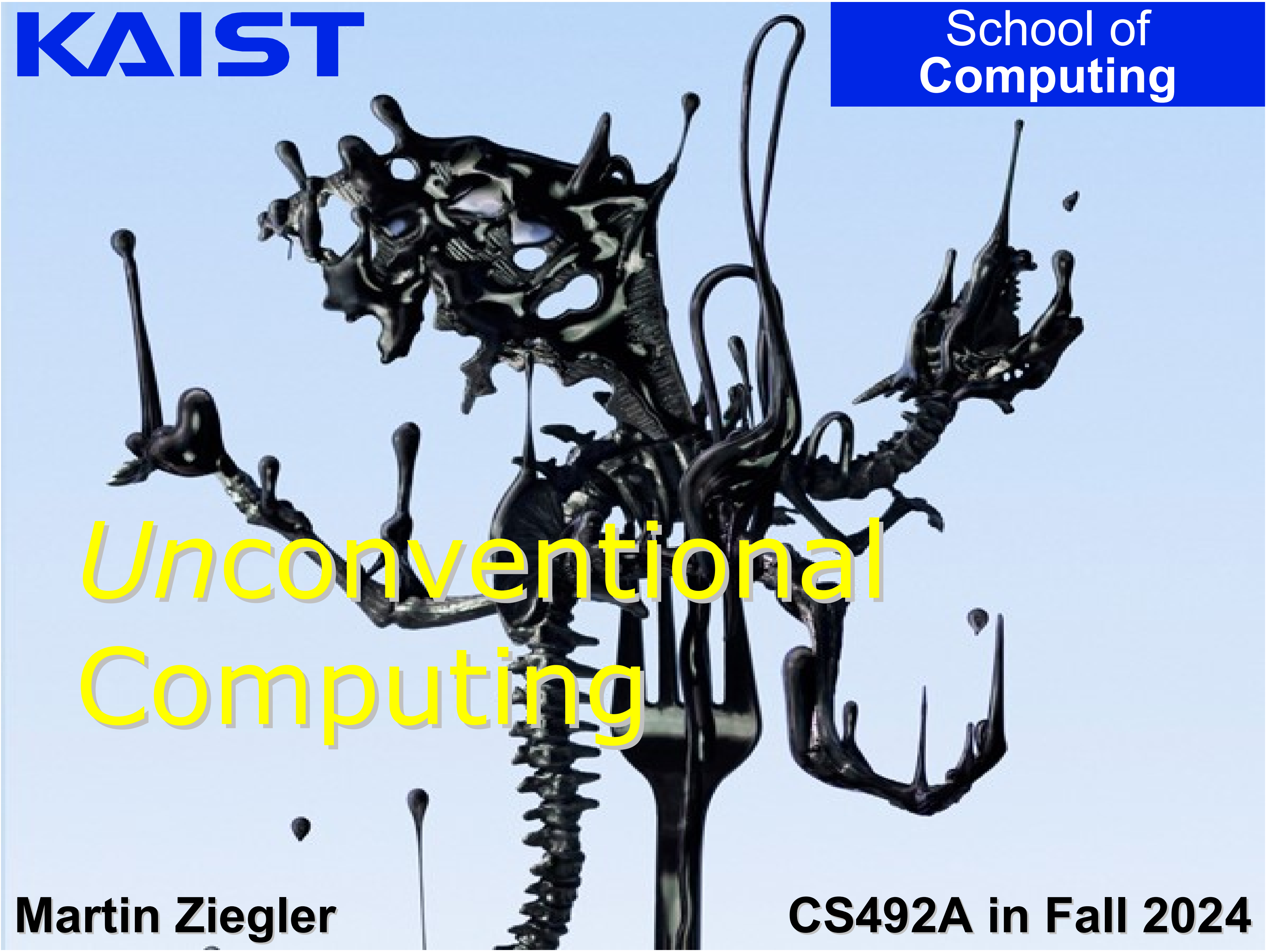


KAIST

School of **Computing**

*Un*conventional Computing

Martin Ziegler

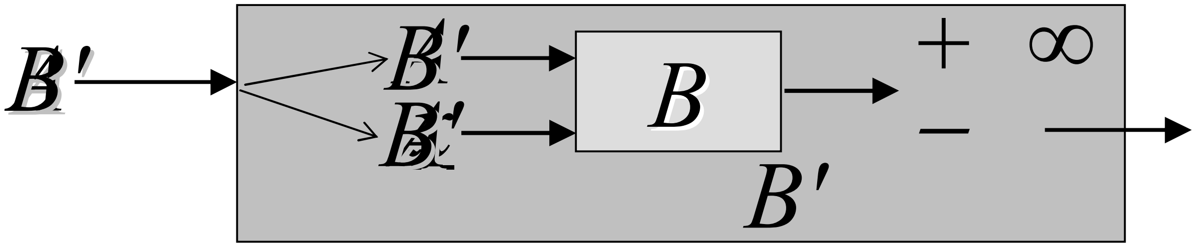
CS492A in Fall 2024

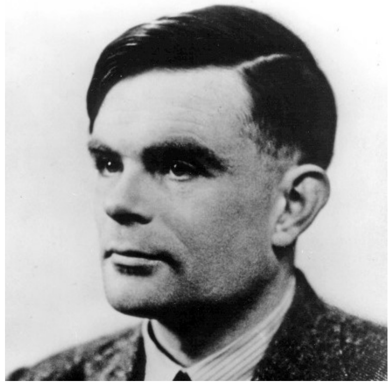
§2 *Asymptotic* (≈∞) Computing

- Motivation/Recap: Halting Problem
- (Relativistic) Zeno Machines
- Revising Computation, Shoenfield Lemma
- *Mechanical* Asymptotic Computation
- *Geometric Optical* Asymptotic Computation
  [**doi:10.1007/BF02574009**]
- (*Analog* Zeno Machines: §3)

Alan M. Turing 1936

- first scientific calculations on digital computers
- *What are its fundamental limitations?*





- Undecidable Halting Problem H: **No algorithm** B can always correctly ans ~~simulator/interpreter B~~ ?
Given $\langle A,\underline{x}\rangle$*, does algorithm A terminate on input* $\underline{x}$*?*

Proof by contradiction: Consider algorithm B' that, on input A, executes B on $\langle A,A\rangle$ and, upon a positive answer, loops infinitely. How does B' behave on B' ?

Approximating the "Halting Problem"

∃**?** algorithm which, on input $\langle\mathcal{A},x\rangle\in\mathbb{N}$,

i) stops with answer "**Yes**" in case $\langle\mathcal{A},x\rangle\in H$ and doesn't answer/stop in case $\langle\mathcal{A},x\rangle\notin H$ √

ii) stops with answer "**No**" in case $\langle\mathcal{A},x\rangle\notin H$ and doesn't answer/stop in case $\langle\mathcal{A},x\rangle\in H$ ↯

iii) always stops&answers **always** *correct* ↯

v) always stops&answers, **in**finitely often *correct* √

vi) always stops&answers, only **finitely** often ***in***correct ↯

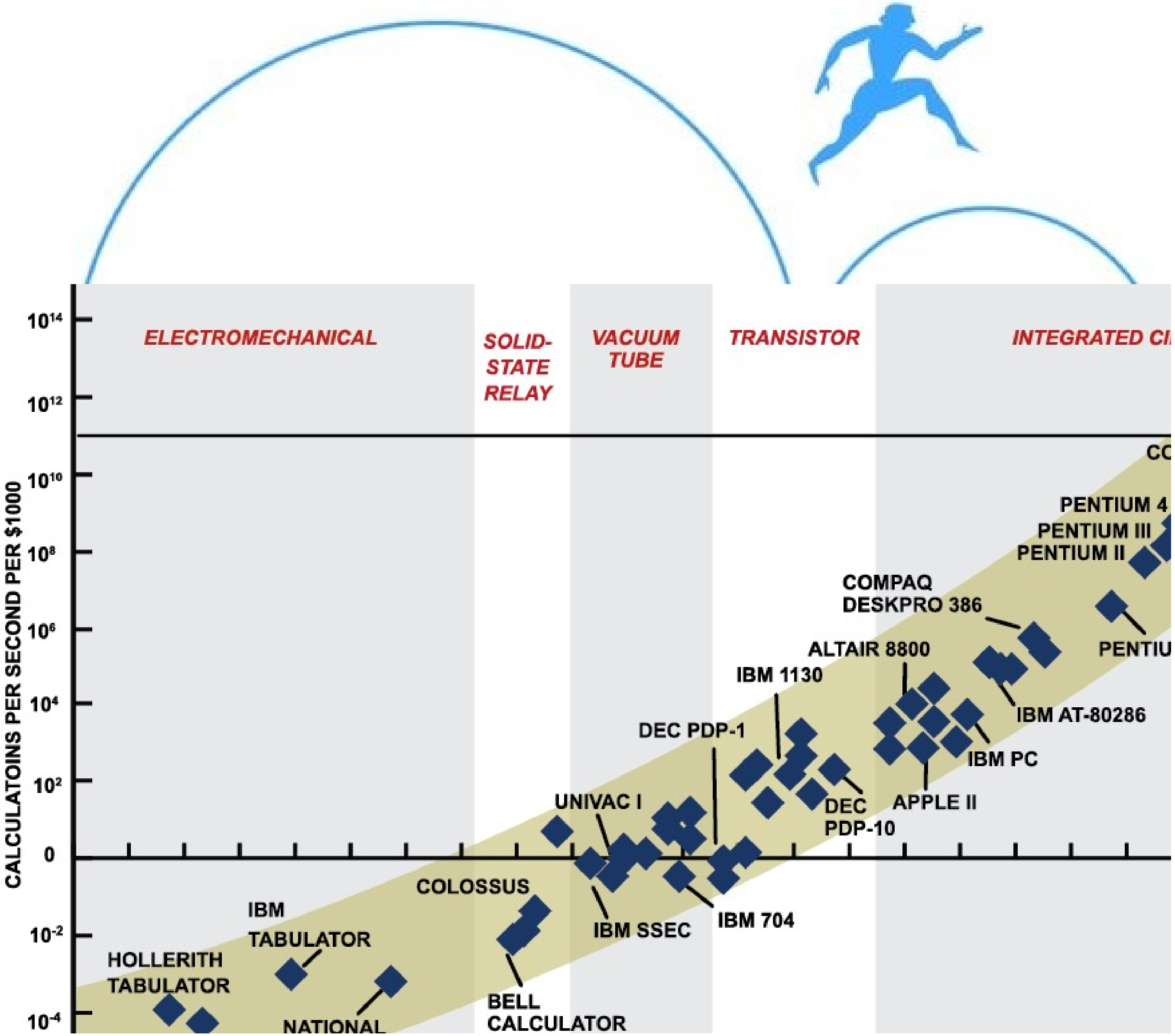
any non-trivial Markov property of a program (Rice Theorem)

$$H = \{\langle\mathcal{A},x\rangle : \mathcal{A} \text{ terminates on input } x\} \subseteq \mathbb{N}$$

§2 Accelerating/Zeno Machines

*in*finite #steps
in finite time,
asymptotically

accelerating computation

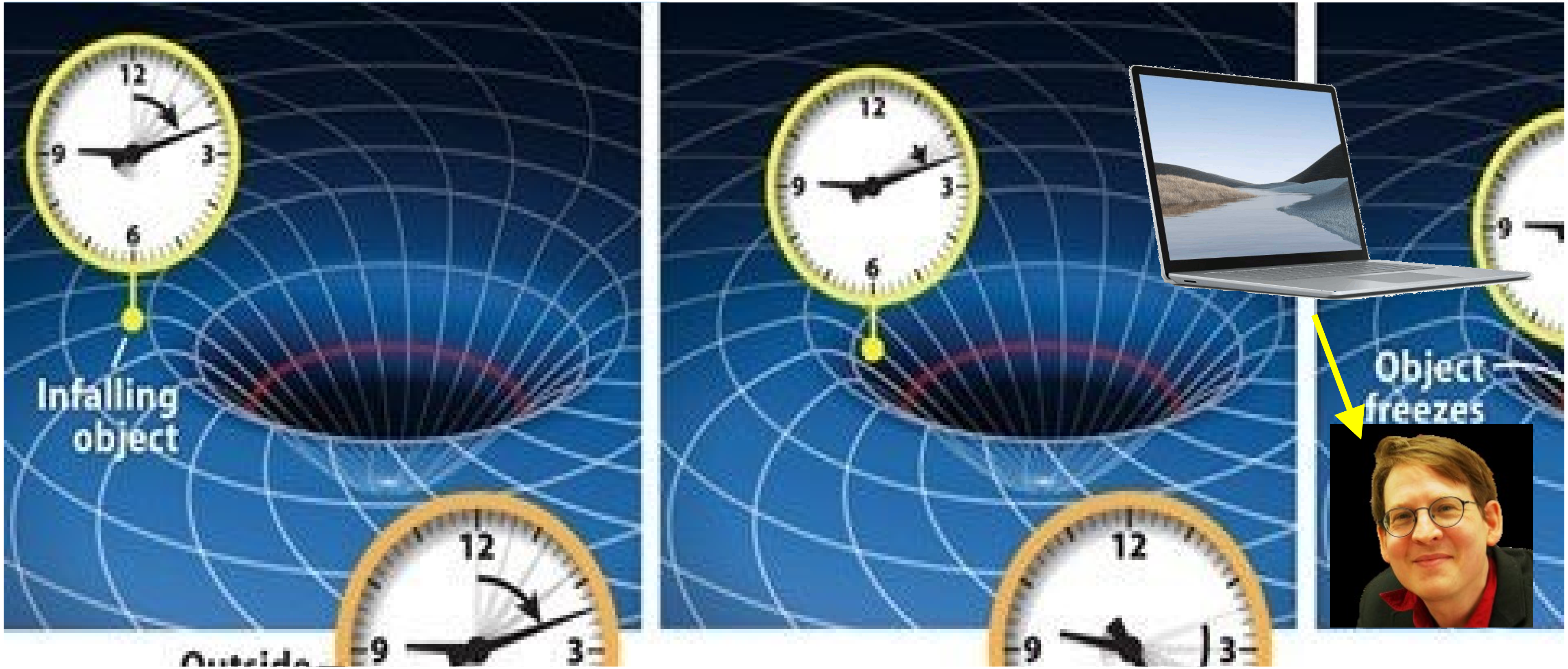


Recall:
Current PC only *approximates* a Turing machine with ∞ tapes...

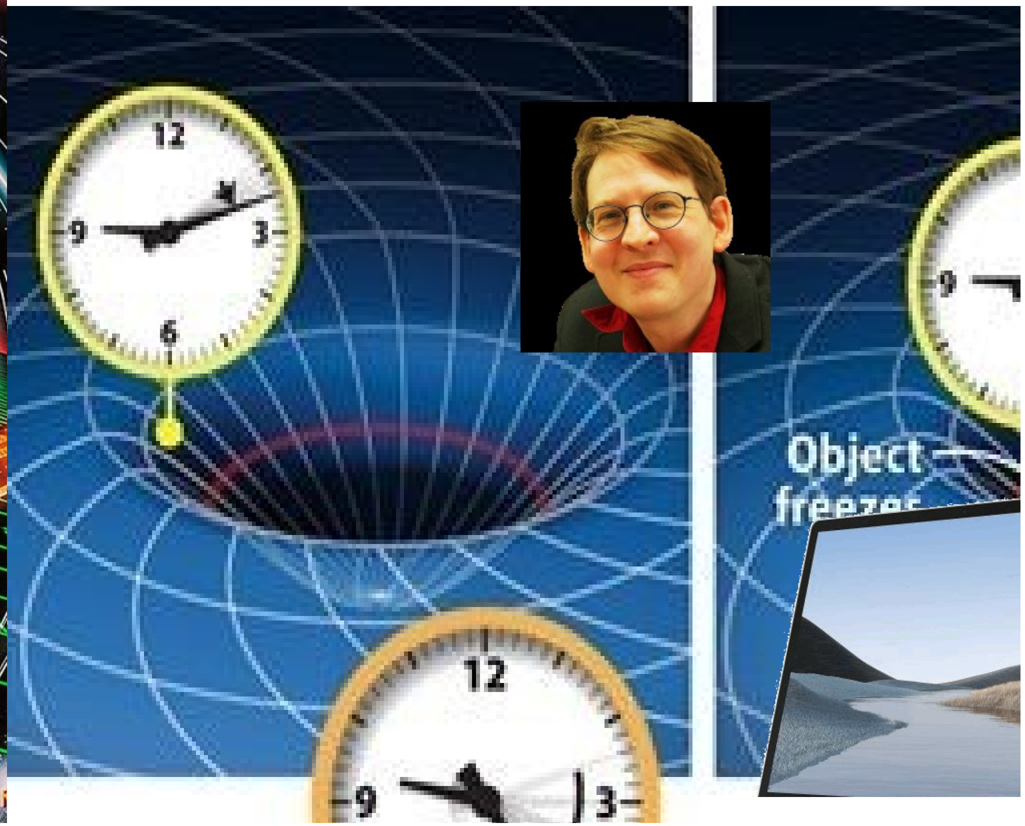
§2 *Relativistic* Zeno Machines

*in*finite #steps
in finite time

"accelerating computation"



§2 *Relativistic* Zeno Machines



§2 Terminating Computation

∃? algorithm which, on input $\langle\mathcal{A},x\rangle\in\mathbb{N}$,

i) stops with answer "**Yes**" in case $\langle\mathcal{A},x\rangle\in H$
and doesn't answer/stop in case $\langle\mathcal{A},x\rangle\notin H$ √

ii) stops with answer "**No**" in case $\langle\mathcal{A},x\rangle\notin H$
and doesn't answer/stop in case $\langle\mathcal{A},x\rangle\in H$ ↯

iii) always stops&answers **always** *correct* ↯

v) always stops&answers,
infinitely often *correct* √

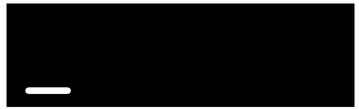
vi) always stops&answers,
only **finitely** often ***in***correct ↯

$H = \{\langle\mathcal{A},x\rangle : \mathcal{A} \text{ terminates on input } x\} \subseteq \mathbb{N}$

§2 *Revising* Computation

Algorithms which (produce output but which) **don't** stop

Output may be *revised*:



a) ~~precisely once~~

b) at most once e.g. H, $\bar{H}$

c) ~~precisely twice~~

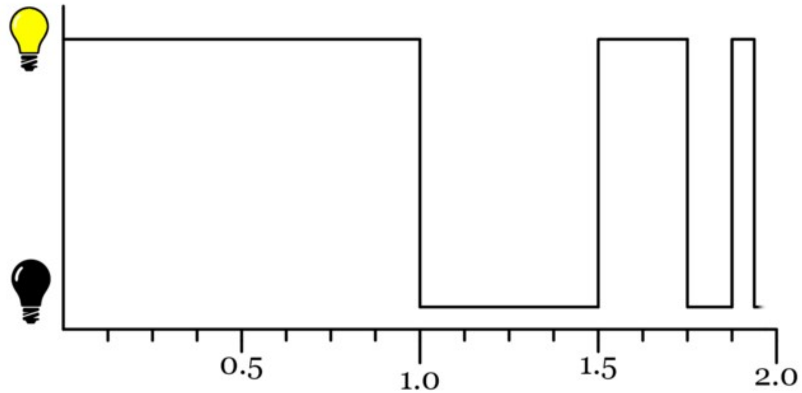
d) at most twice

e) ...

f) finitely often

g) ~~infinitely often~~

always stops&answers



Thomson's Lamp

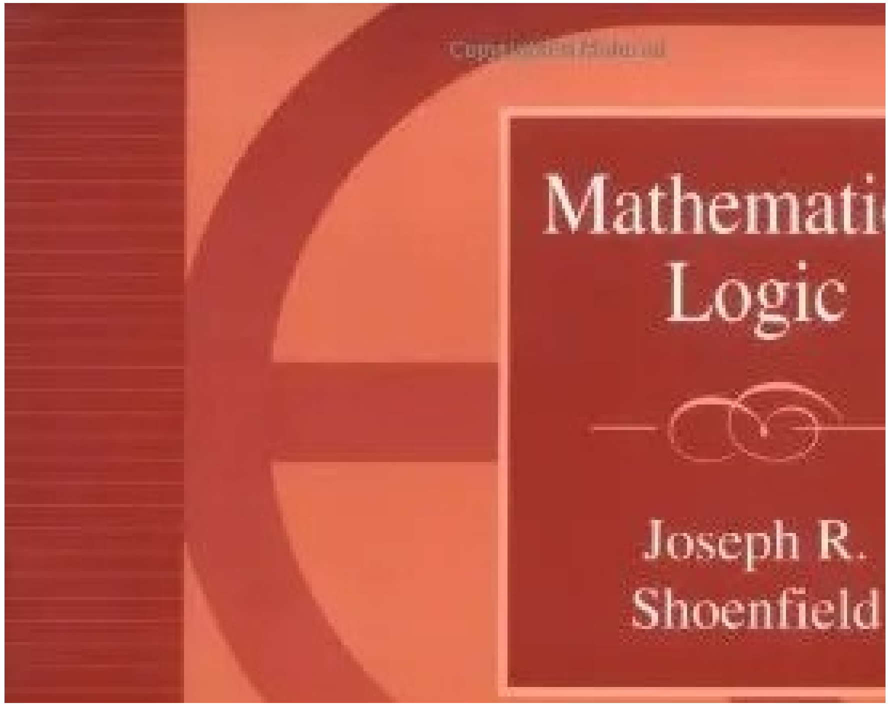
$$H \;=\; \{\, \langle \mathcal{A}, x \rangle : \mathcal{A} \text{ terminates on input } x \,\} \quad \subseteq \mathbb{N}$$

§2 *Shoenfield* Lemma

Algorithms which (produce output but which) **don't** stop

Output may be *revised*:

a) ~~precisely once~~
b) at most once
c) ~~precisely twice~~
d) at most twice
e) ...
f) finitely often

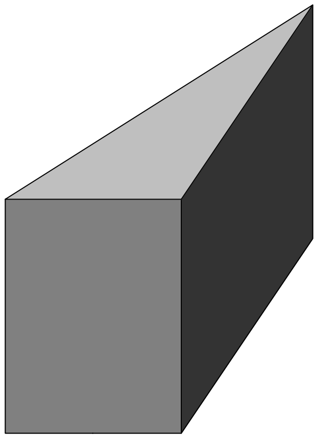


Lemma (Joseph Shoenfield): A *non*-stopping algorithm, allowed to *revise* its output $\leq k$ times, is equivalent to a stopping algorithm allowed k queries to oracle H.

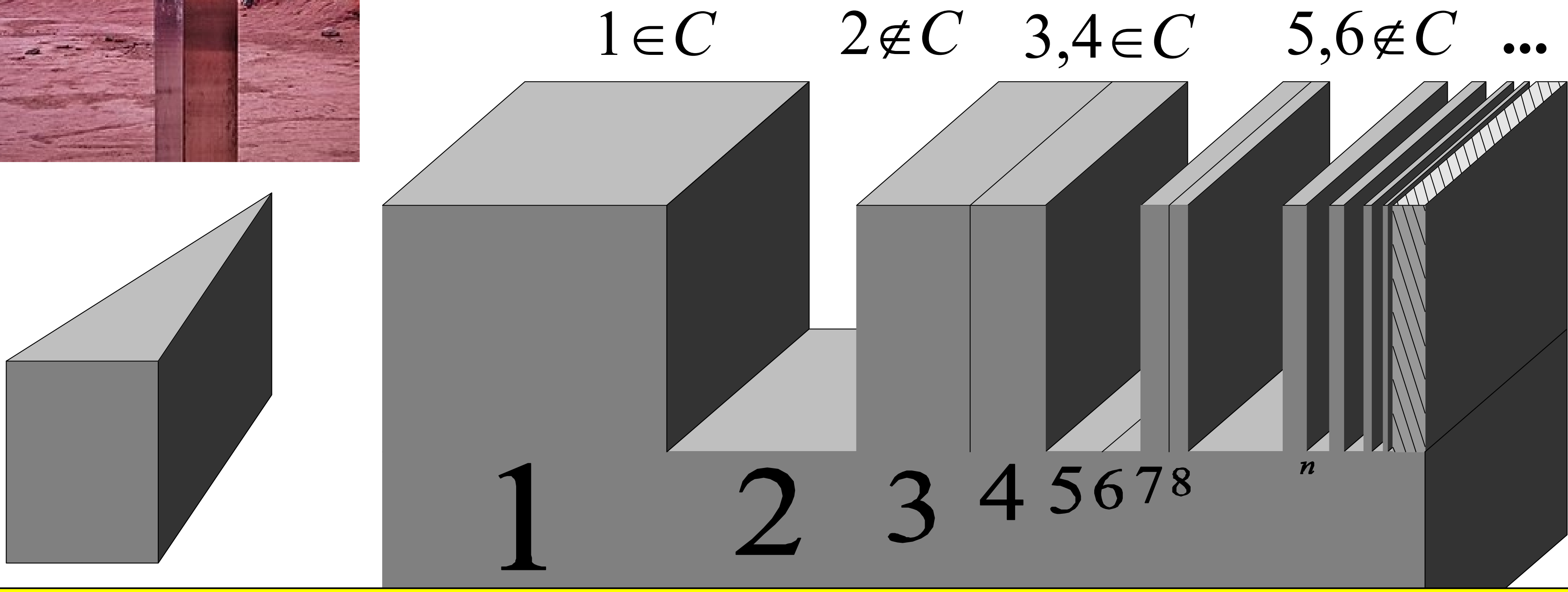
$H = \{ \langle \mathcal{A}, x \rangle : \mathcal{A}$ terminates on input $x \} \subseteq \mathbb{N}$

§2 *Finite* Mechanical Comb Oracle

$1 \in C$ $2 \notin C$ $3,4 \in C$ $5,6 \notin C$ ...



§2 Zeno's Comb as Mechanical Oracle



$C := H = \{ \langle \mathcal{A},x \rangle : \mathcal{A}$ terminates on input $x \} \subseteq \mathbb{N}$

§2 *Un/computable* Real Constants

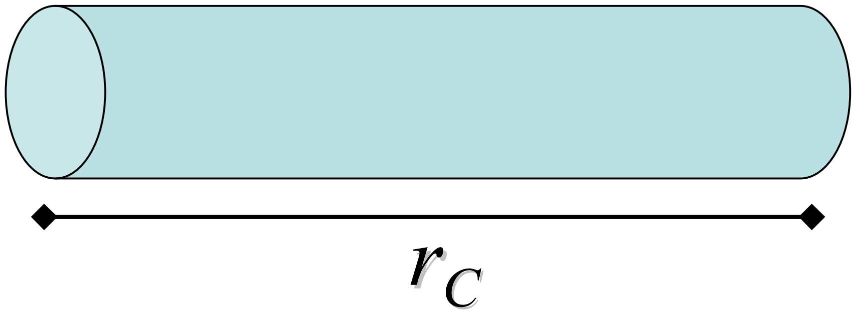
Definition: Call $r \in \mathbb{R}$ *computable*
if r has a decidable binary expansion.

Fact: a) Every $r \in \mathbb{Q}$ is *computable.*

b) If r is computable, then so are: $\sin(r)$, $\cos(r)$, $\operatorname{atan}(r)$, ...

$1 \in C$ $2 \notin C$ $3,4 \in C$ $5,6 \notin C$ ...

$r_C :=$
$\sum_{n \in C} 2^{-n}$

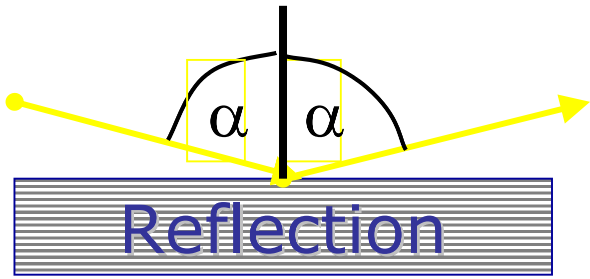


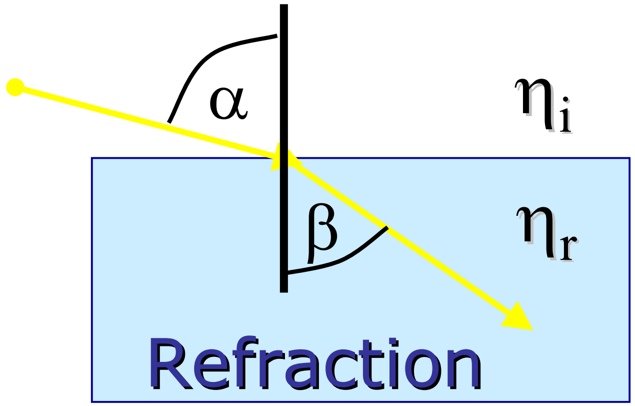
$C := H = \{ \langle \mathcal{A}, x \rangle : \mathcal{A}$ terminates on input $x \} \subseteq \mathbb{N}$

§2 *Geometric* Optics (*reversible*)

No *aberrations*:
- chromatic
- defocus/ attenuation
- coma
- wave/particle

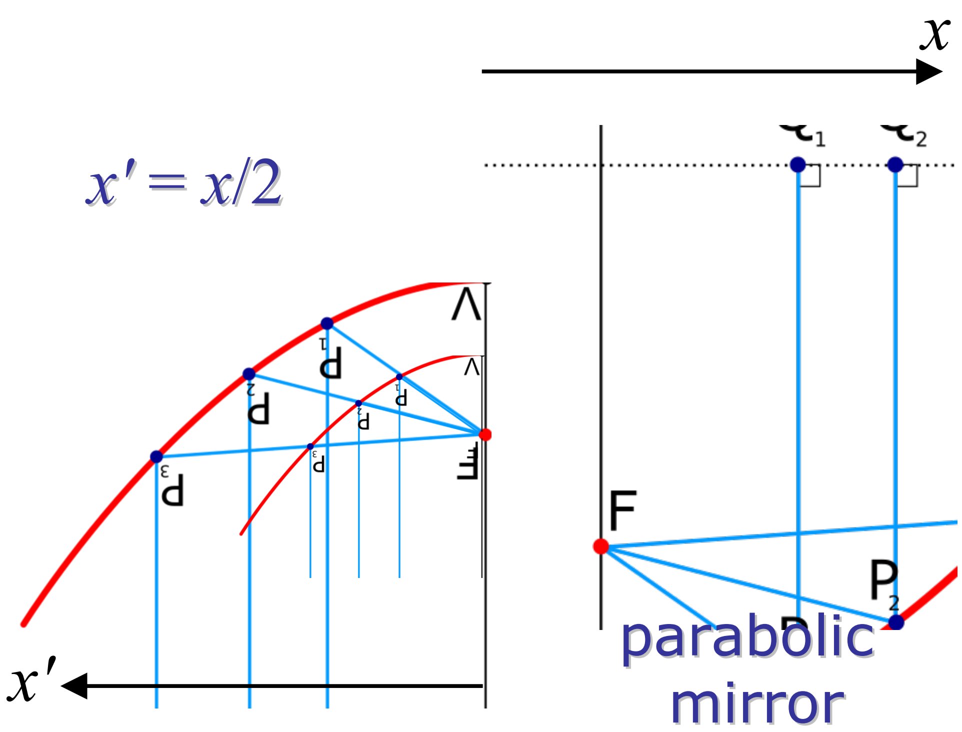
No un*computable* coordinates /directions!





$$\eta_i \cdot \sin\alpha = \eta_r \cdot \sin\beta$$

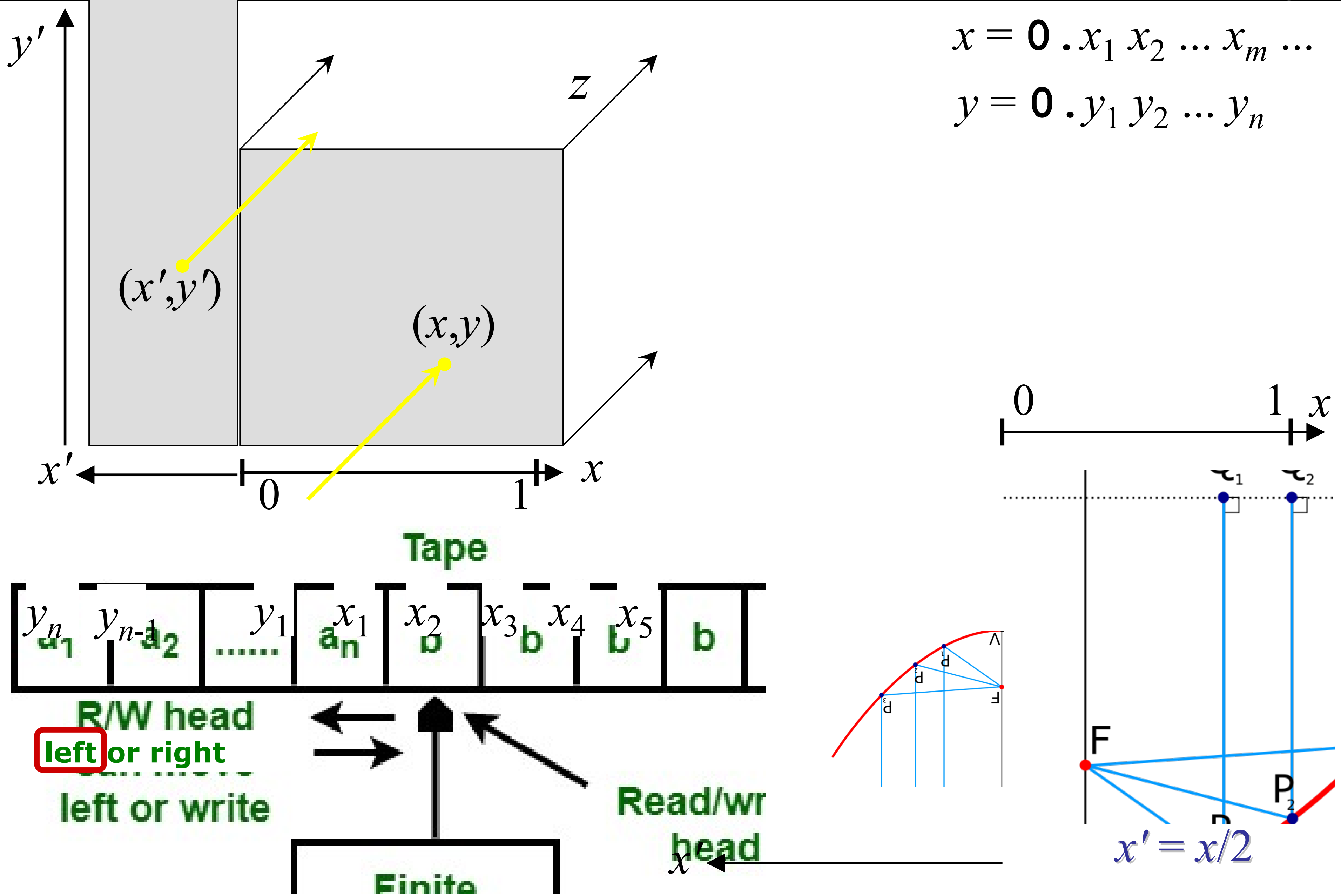
(Snell's Law)



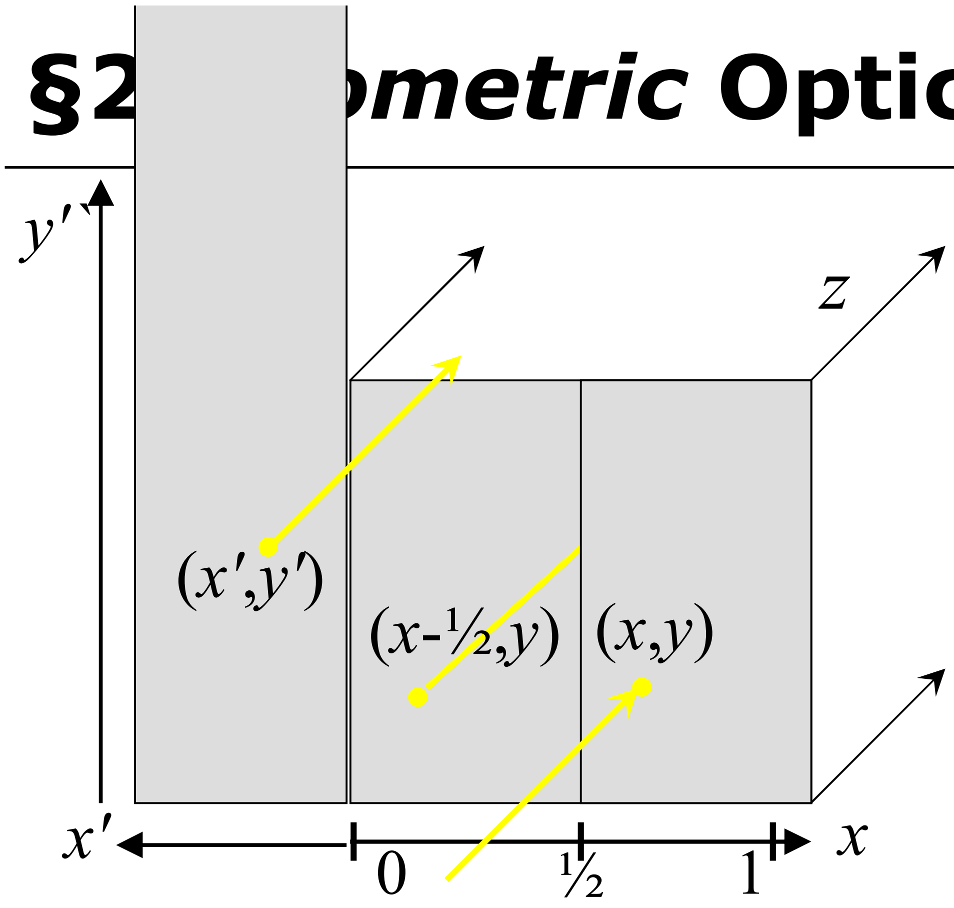
§2 …ometric Optics (reversible)

$$x = 0\,.\,x_1\,x_2\,\dots\,x_m\,\dots$$

$$y = 0\,.\,y_1\,y_2\,\dots\,y_n$$

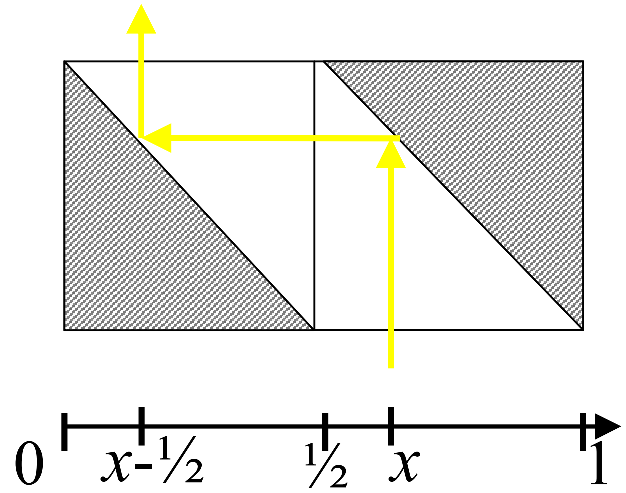


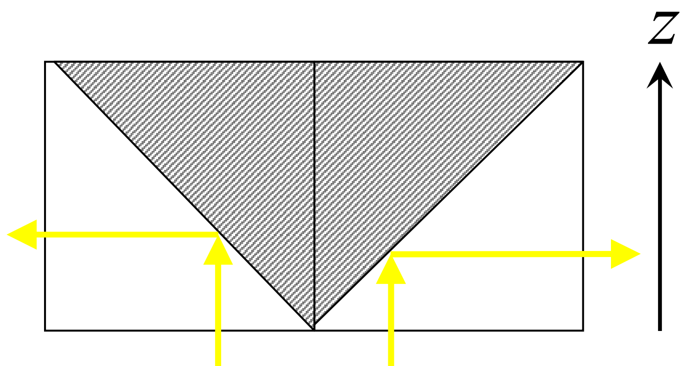
§2 ometric Optics (*reversible*)

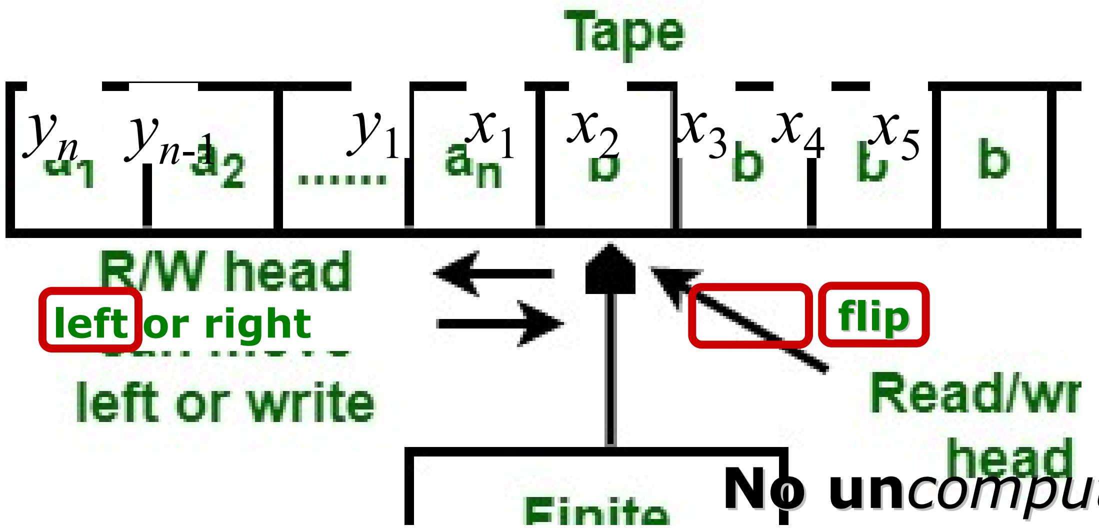


$x = 0\,.\,x_1\,x_2\,\ldots\,x_m\,\ldots$

$y = 0\,.\,y_1\,y_2\,\ldots\,y_n$



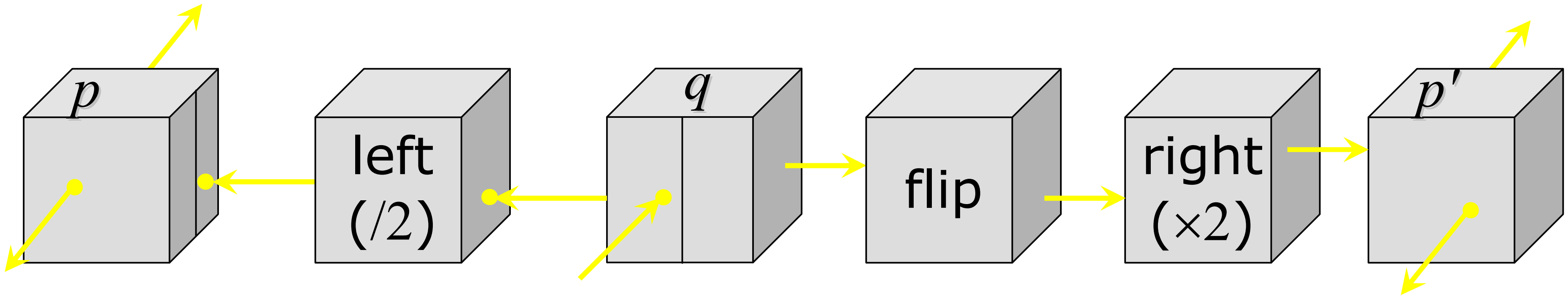




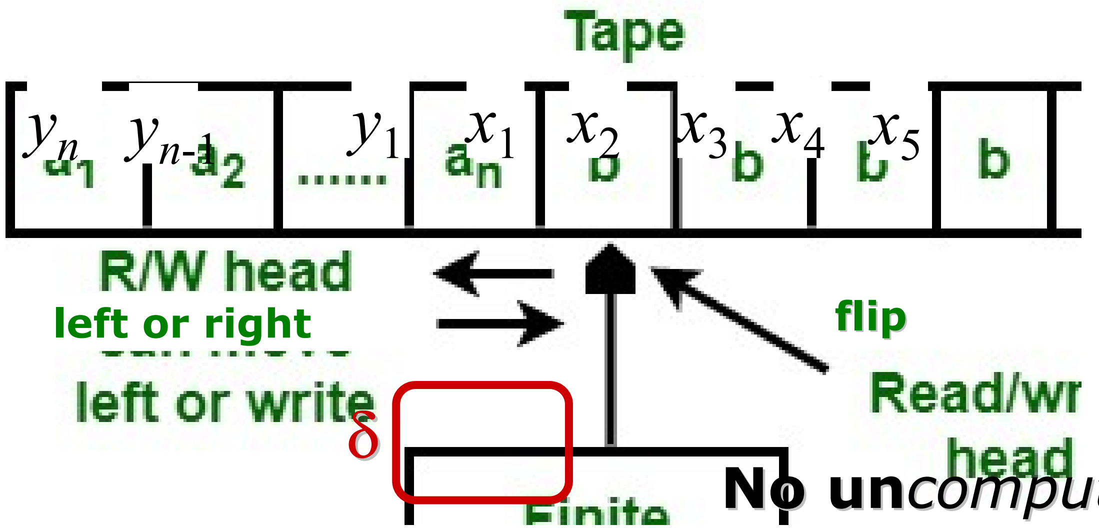
No un*computable* coordinates/directions!

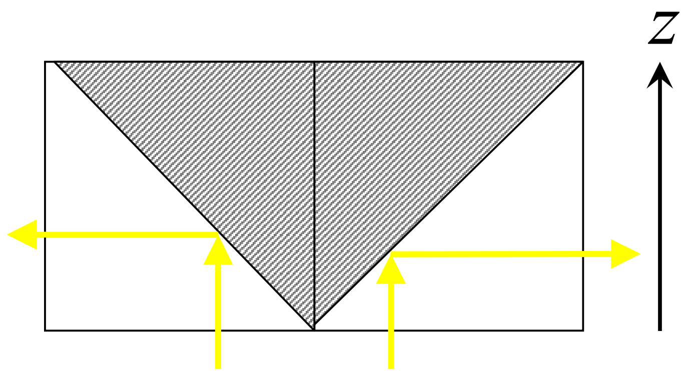
§2 *Geometric* Optics (*reversible*)

Fact: Every Turing machine can be made reversible!



$\delta(q,\mathtt{0}) = (\mathtt{0},\mathtt{L},p)$ $\delta(q,\mathtt{1}) = (\mathtt{0},\mathtt{R},p')$



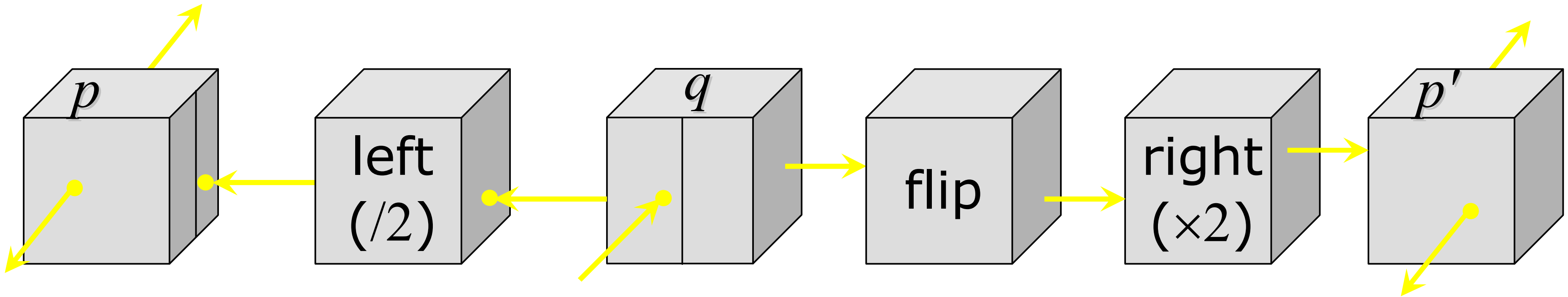


No un*computable* coordinates/directions!

"Computability and Complexity of Ray Tracing"

John H. Reif, J. Doug Tygar, Akitoshi Yoshida

Discrete&Computational Geometry 11 (1994)



Questions: **a)** How does this *Geometric Optical Computing* differ from *Photonic Computing*?

b) How much *physical* time does it take asymptotically to simulate a Turing machine making $t(n)$ steps?

c) How much *physical* space does it take asymptotically to simulate a Turing machine using $s(n)$ cells?

d) When is this theoretical simulation im/practical?

§2 *Asymptotic* (≈∞) Computing

- Motivation/Recap: Halting Problem
- (Relativistic) Zeno Machines
- Revising Computation, Shoenfield Lemma
- *Mechanical* Asymptotic Computation
- *Geometric Optical* Asymptotic Computation
  [**doi:10.1007/BF02574009**]
- (*Analog* Zeno Machines: §3)