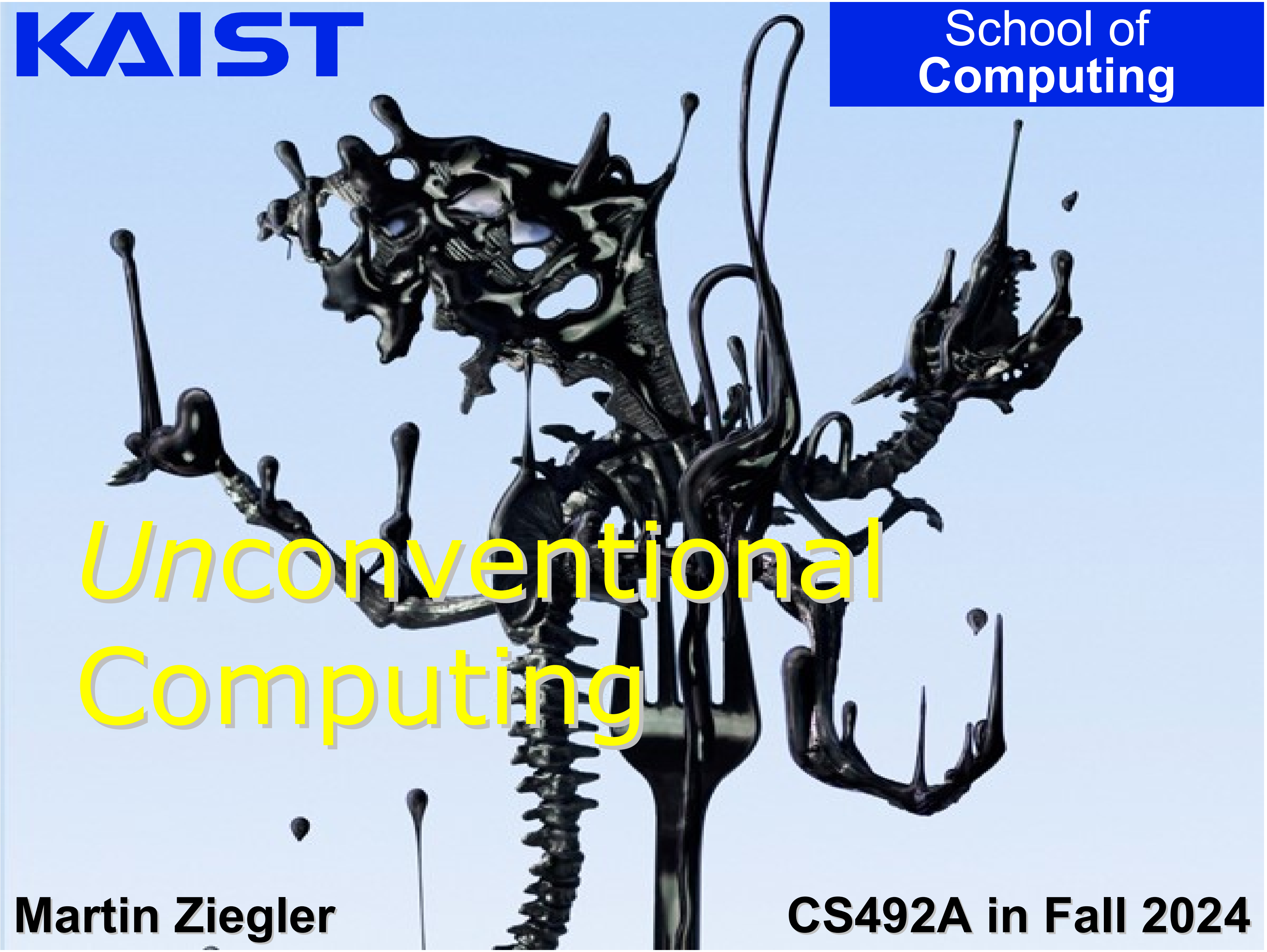
KAIST

School of Computing

*Un*conventional Computing



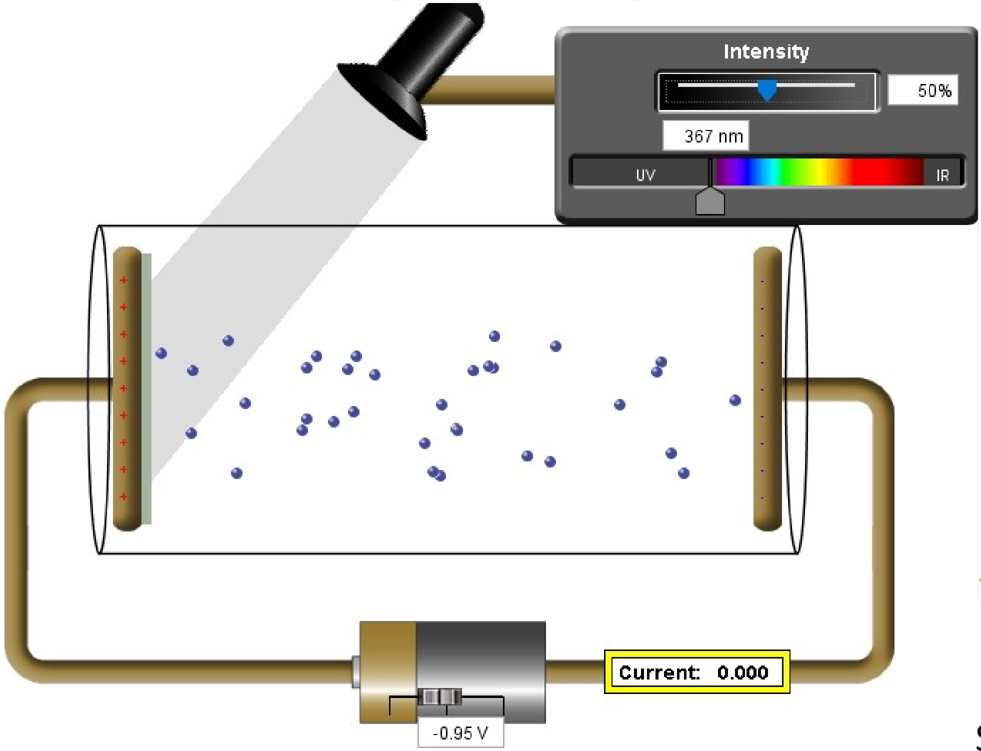
Martin Ziegler

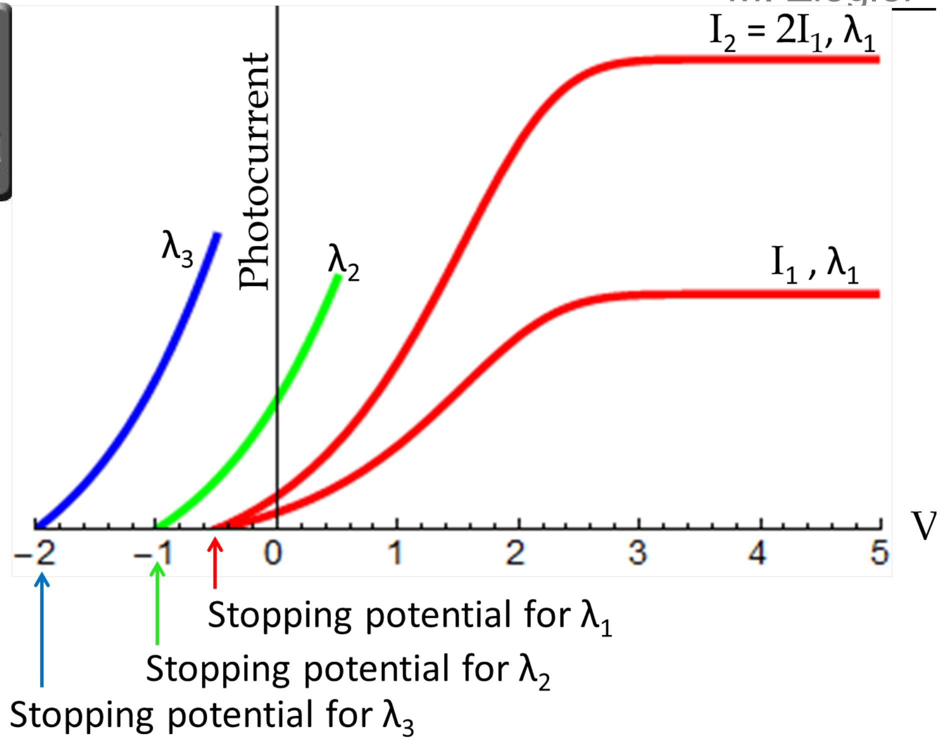
CS492A in Fall 2024

§4 Quantum Computing

- Recap: Experimental Physical Evidence
- Math Background: States and Operators
- Qubits and Primitive Gates
- Quantum Circuit for

§4 Recap: Experimental Physics



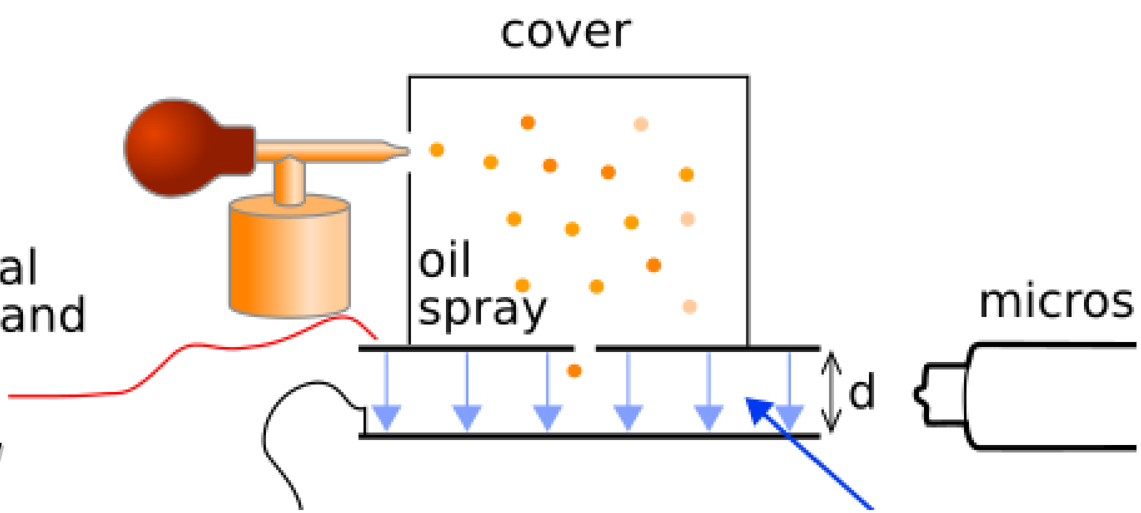


Einstein (1905):

And so is *light!*

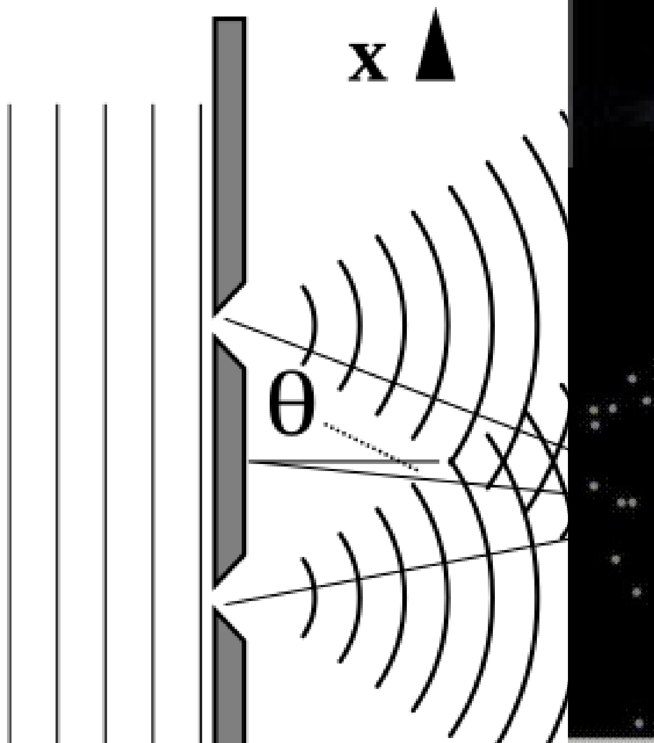
Robert Millikan "Oil drop" (1909):

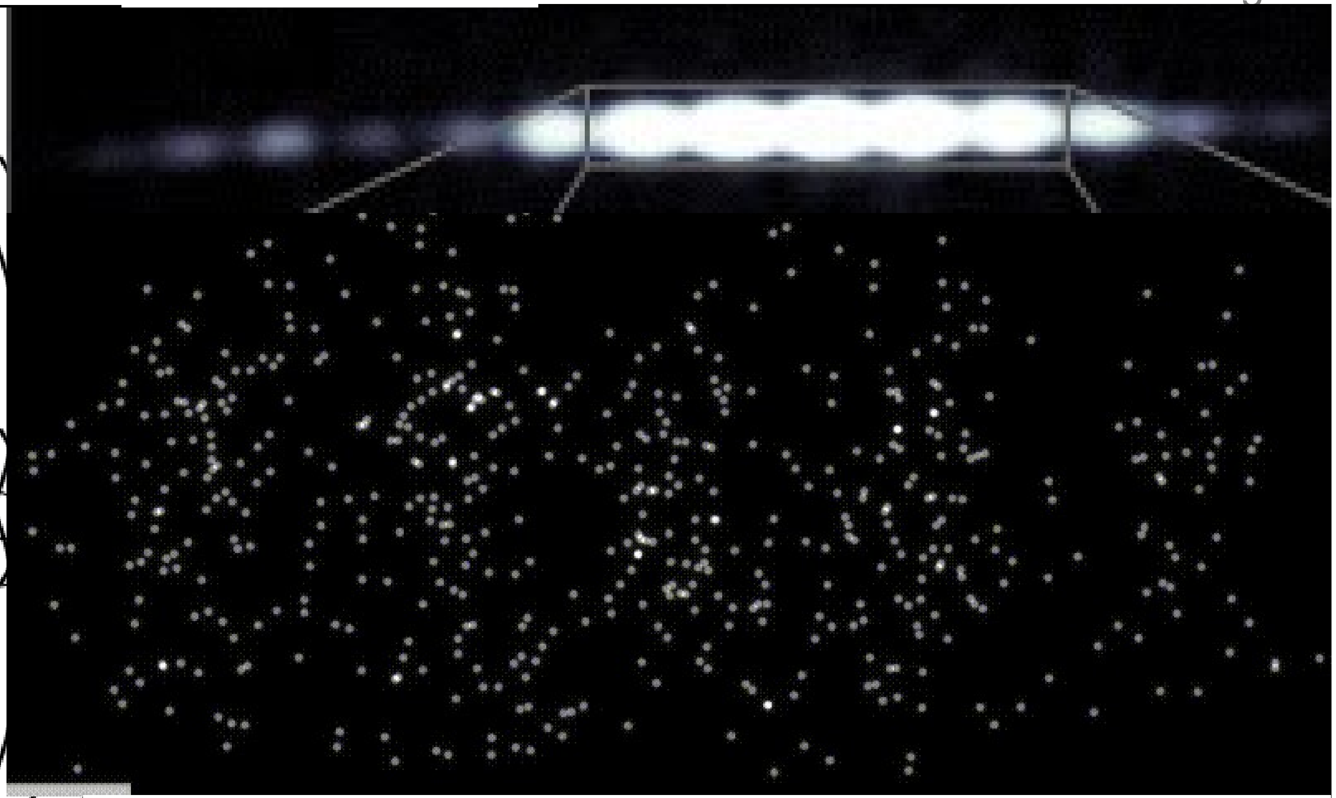
several thousand volts



Electrons are *particles!*

§4 Recap: Experimental Physics





Einstein (1905):

And so is *light*!

Robert Millikan "Oil drop" (1909):

Electrons are *particles*!

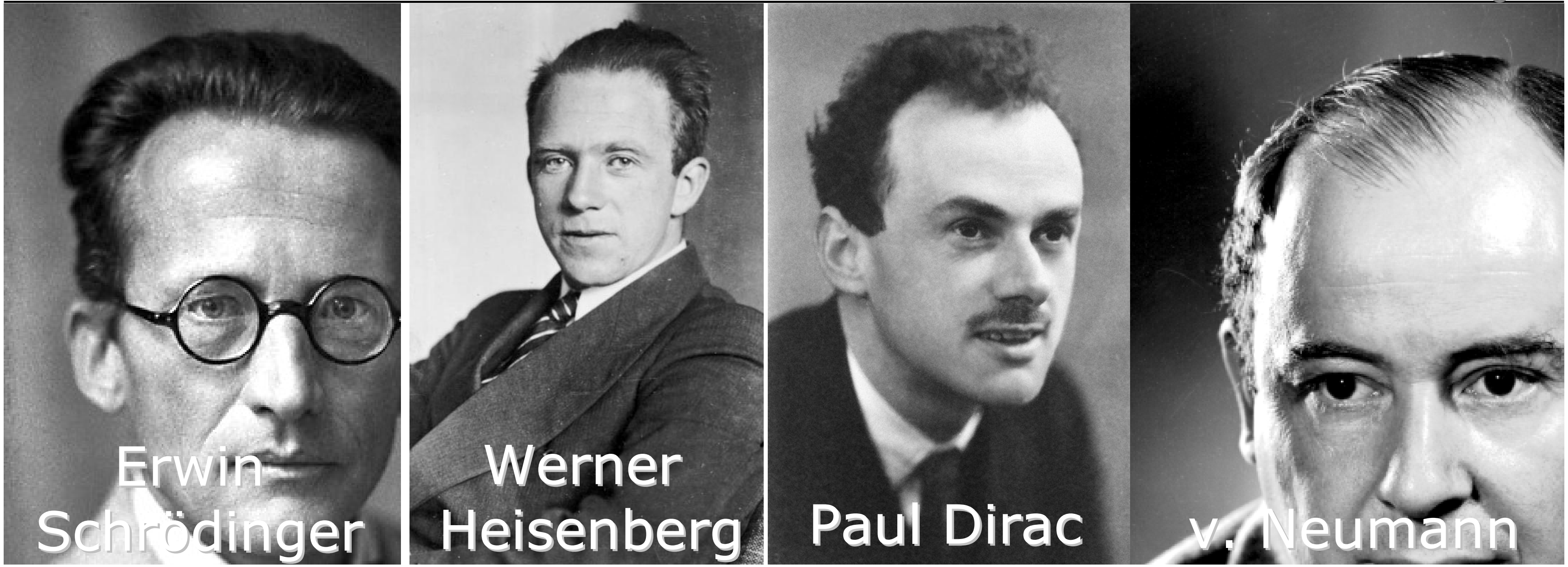
Claus Jönsson (1959):

And so are *electrons!*

Thomas Young (1801):

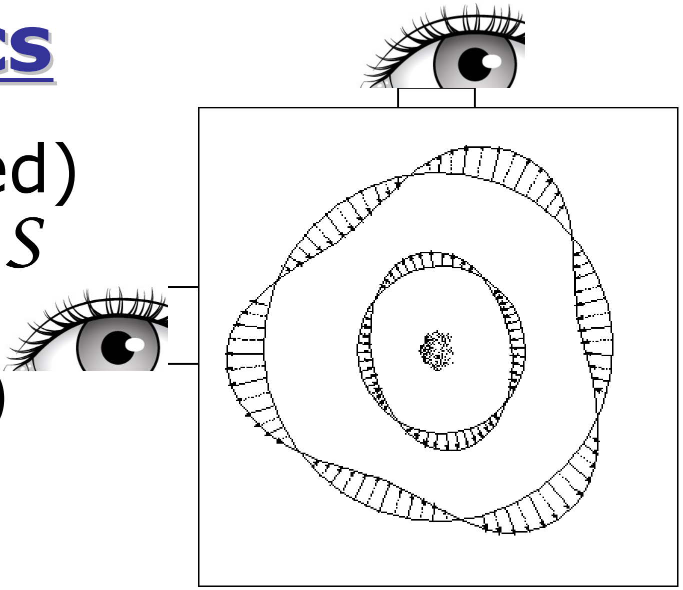
Light is a *wave!*

§4 *Basic* Quantum Mechanics

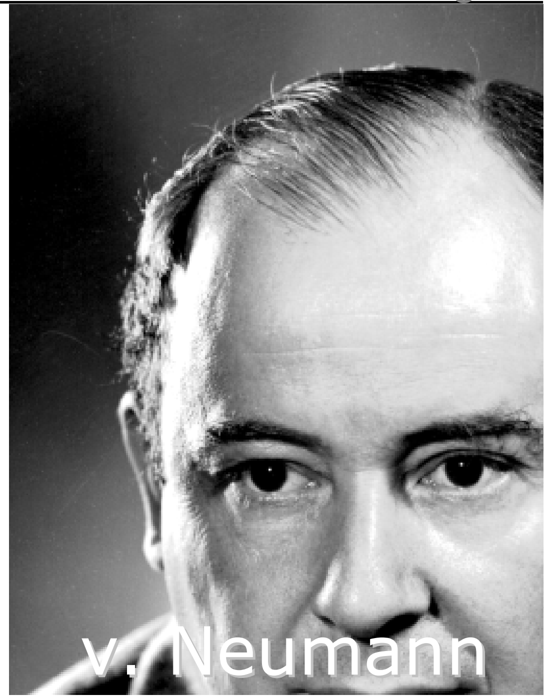


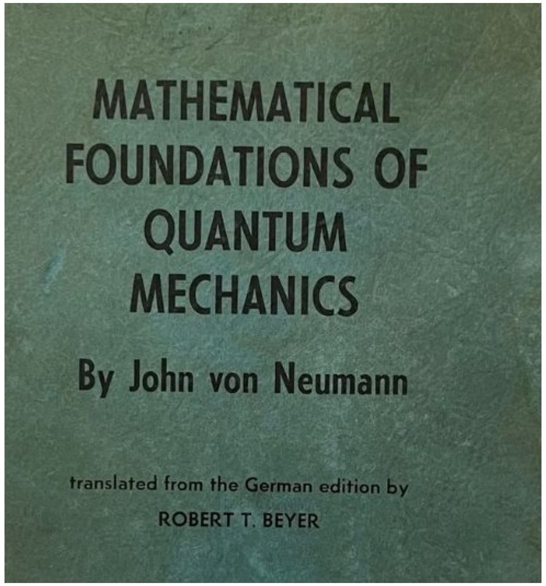
- NOT *Path Integral* (Richard Feynman)
- NOT *Quantum Field Theory*
  (Dyson, Feynman, Schwinger, Tomonaga)
- NOT *Relativistic* Quantum Mechanics

§4 *Math* of Quantum Mechanics



Physics	Math
I (isolated) system $\mathcal{S}$	Hilbert Space $\mathcal{H}$
II (pure) states s, s' of $\mathcal{S}$	normal vectors $\psi, \psi' \in \mathcal{H}$
III observable $\mathcal{A}$, $\mathcal{A}'$ of $\mathcal{S}$	Hermit. operator $\boldsymbol{A}, \boldsymbol{A}'$ on $\mathcal{H}$
IV measurement of $\mathcal{A}$	eigenvalue a of $\boldsymbol{A}$
V time evolution $s(0) \to s(t)$	Schrödinger Eq. $i\hbar\, \mathrm{d}/\mathrm{d}t\, \psi(t) = \boldsymbol{H}\psi(t)$





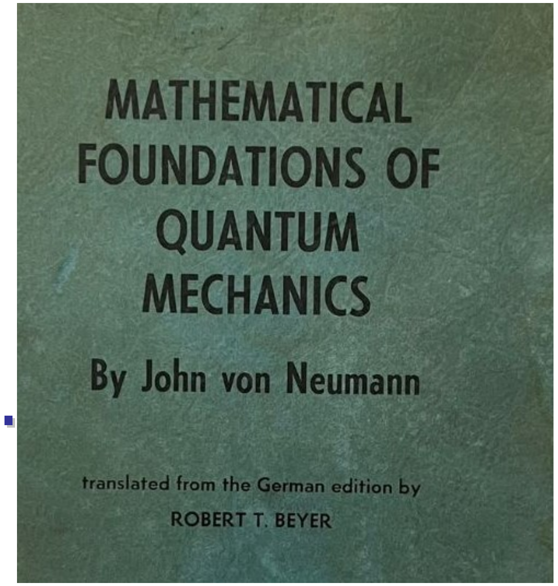
§4 *Axioms* of Quantum Mechanics

Ia. To any (isolated) physical system S corresponds a Hilbert space $\mathcal{H}$ called the *state space*.

Ib. The state space of a system S composed from sub-systems S_j is the *tensor product* $\mathcal{H} = \otimes_j \mathcal{H}_j$ of the state spaces associated with components S_j.

IIa. A *pure* state $s=s(t)$ of S at time t corresponds to a *unit* (=norm**1**) vector $\psi = \psi(t) \in \mathcal{H}$.

IIb. A statistical *ensemble* (=mix) of pure states/vectors s_k/ψ_k with weights $w_k \in [0;1]$ corresponds to a *density* (=pos. semi.trace**1**) *operator* $\rho = \sum_k w_k \cdot |\psi_k\rangle\langle\psi_k|$



§4 *Axioms* of Quantum Mechanics

III. A physical observable $\mathcal{A}$ on $\mathcal{S}$ corresponds to a Hermitian operator $\boldsymbol{A}$ on $\mathcal{H}$. Performing a measurement of $\mathcal{A}$ will produce **some** eigenvalue of $\boldsymbol{A}$.

IVa. When $\mathcal{S}$ is in *pure* state ψ, measuring $\mathcal{A}$ produces eigenvalue a with **probability** $|\langle\psi_a|\psi\rangle|^2$, where ψ_a is a unit eigenvector of $\boldsymbol{A}$ to eigenvalue a — in case a is discrete non-degenerate.

Other formula when a is degenerate/ continuous

IVb. When $\mathcal{S}$ is in *mixed* state with density $\boldsymbol{\rho}$, measuring $\mathcal{A}$ produces eigenvalue a with **probability** $\langle\psi_a|\,\boldsymbol{\rho}\,\psi_a\rangle$, where ψ_a is a unit eigenvector of $\boldsymbol{A}$ to eigenvalue a — in case a is discrete non-degenerate.

§4 *Axioms* of Quantum Mechanics

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Hamilton operator / observable "*energy*" **(III)**

IIa. A *pure* state $s=s(t)$ of $\mathcal{S}$ at time t corresp to a *unit* (=norm**1**) vector $\psi = \psi(t) \in \mathcal{H}$.

V. The time evolution of a *pure* state $\psi(0) \rightarrow \psi(t) \in \mathcal{H}$ is described by Schrödinger's Equ.n: $i\hbar\, \mathrm{d}/\mathrm{d}t\, \psi(t) = \boldsymbol{H}\, \psi(t)$

§4 Qubits and Quantum Gates

Example: $\mathcal{H}_j = \mathbb{C}^2$ (qubit),
ortho-basis $(0,1) =: |\mathbf{0}\rangle$ and $(1,0) =: |\mathbf{1}\rangle$

$\otimes^n \mathbb{C}^2$ (n qubit**s**) has dimension 2^n
with ortho-basis $|\mathbf{0...0}\rangle$ … $|\mathbf{1...1}\rangle$

Recall Ib. The state space of a system S composed from sub-systems S_j is the *tensor product* $\mathcal{H} = \otimes_j \mathcal{H}_j$ of the state spaces associated with components S_j.